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Mathematics

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Contents

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This publication is available on the Ministry of Education's website, at http://www.edu.gov.on.ca.

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

Introduction

This document replaces *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*. Beginning in September 2005, all mathematics programs for Grades 1 to 8 will be based on the expectations outlined in this document.

The Importance of Mathematics

An information- and technology-based society requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. The Ontario mathematics curriculum for Grades 1 to 8 provides the framework needed to meet these goals.

Learning mathematics results in more than a mastery of basic skills. It equips students with a concise and powerful means of communication. Mathematical structures, operations, processes, and language provide students with a framework and tools for reasoning, justifying conclusions, and expressing ideas clearly. Through mathematical activities that are practical and relevant to their lives, students develop mathematical understanding, problem-solving skills, and related technological skills that they can apply in their daily lives and, eventually, in the workplace.

Mathematics is a powerful learning tool. As students identify relationships between mathematical concepts and everyday situations and make connections between mathematics and other subjects, they develop the ability to use mathematics to extend and apply their knowledge in other curriculum areas, including science, music, and language.

Principles Underlying the Ontario Mathematics Curriculum

This curriculum recognizes the diversity that exists among students who study mathematics. It is based on the belief that all students can learn mathematics and deserve the opportunity to do so. It recognizes that all students do not necessarily learn mathematics in the same way, using the same resources, and within the same time frames. It supports equity by promoting the active participation of all students and by clearly identifying the knowledge and skills students are expected to demonstrate in every grade. It recognizes different learning styles and sets expectations that call for the use of a variety of instructional and assessment tools and strategies. It aims to challenge all students by including expectations that require them to use higher-order thinking skills and to make connections between related mathematical concepts and between mathematics, other disciplines, and the real world.

This curriculum is designed to help students build the solid conceptual foundation in mathematics that will enable them to apply their knowledge and further their learning successfully. It is based on the belief that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving and are then guided carefully into an understanding of the mathematical principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum.

Attention to the *processes* that support effective learning of mathematics is also considered to be essential to a balanced mathematics program. Seven mathematical processes are identified in this curriculum document: *problem solving*, *reasoning and proving*, *reflecting*, *selecting tools and computational strategies*, *connecting*, *representing*, and *communicating*. The curriculum for each grade outlined in this document includes a set of "mathematical process expectations" that describe the practices students need to learn and apply in all areas of their study of mathematics.

This curriculum recognizes the benefits that current technologies can bring to the learning and doing of mathematics. It therefore integrates the use of appropriate technologies, while recognizing the continuing importance of students' mastering essential arithmetic skills.

The development of mathematical knowledge is a gradual process. A continuous, cohesive progam throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way. The fundamentals of important concepts, processes, skills, and attitudes are introduced in the primary grades and fostered through the junior and intermediate grades. The program is continuous, as well, from the elementary to the secondary level.

The transition from elementary school mathematics to secondary school mathematics is very important for students' development of confidence and competence. The Grade 9 courses in the Ontario mathematics curriculum build on the knowledge of concepts and the skills that students are expected to have by the end of Grade 8. The strands used are similar to those used in the elementary program, with adjustments made to reflect the more abstract nature of mathematics at the secondary level. Finally, the mathematics courses offered in secondary school are based on principles that are consistent with those that underpin the elementary program, a feature that is essential in facilitating the transition.

Roles and Responsibilities in Mathematics Education

Students. Students have many responsibilities with regard to their learning, and these increase as they advance through elementary and secondary school. Students who are willing to make the effort required and who are able to apply themselves will soon discover that there is a direct relationship between this effort and their achievement in mathematics. There will be some students, however, who will find it more difficult to take responsibility for their learning because of special challenges they face. For these students, the attention, patience, and encouragement of teachers and family can be extremely important factors for success. However, taking responsibility for their own progress and learning is an important part of education for all students.

Understanding mathematical concepts and developing skills in mathematics require a sincere commitment to learning. Younger students must bring a willingness to engage in learning activities and to reflect on their experiences. For older students, the commitment to learning requires an appropriate degree of work and study. Students are expected to learn and apply strategies and processes that promote understanding of concepts and facilitate the application of important skills. Students are also encouraged to pursue opportunities outside the classroom to extend and enrich their understanding of mathematics.

Parents. Parents have an important role to play in supporting student learning. Studies show that students perform better in school if their parents or guardians are involved in their education. By becoming familiar with the curriculum, parents can find out what is being taught in each grade and what their child is expected to learn. This awareness will enhance parents' ability to discuss schoolwork with their child, to communicate with teachers, and to ask relevant questions about their child's progress. Knowledge of the expectations in the various grades also helps parents to interpret their child's report card and to work with teachers to improve their child's learning.

There are other effective ways in which parents can support students' learning. Attending parent-teacher interviews, participating in parent workshops and school council activities (including becoming a school council member), and encouraging students to complete their assignments at home are just a few examples. The mathematics curriculum has the potential to stimulate interest in lifelong learning not only for students but also for their parents and all those with an interest in education.

Teachers. Teachers and students have complementary responsibilities. Teachers are responsible for developing appropriate instructional strategies to help students achieve the curriculum expectations, and for developing appropriate methods for assessing and evaluating student learning. Teachers also support students in developing the reading, writing, and oral communication skills needed for success in learning mathematics. Teachers bring enthusiasm and varied teaching and assessment approaches to the classroom, addressing different student needs and ensuring sound learning opportunities for every student.

Recognizing that students need a solid conceptual foundation in mathematics in order to further develop and apply their knowledge effectively, teachers endeavour to create a classroom environment that engages students' interest and helps them arrive at the understanding of mathematics that is critical to further learning.

It is important for teachers to use a variety of instructional, assessment, and evaluation strategies, in order to provide numerous opportunities for students to develop their ability to solve problems, reason mathematically, and connect the mathematics they are learning to the real world around them. Opportunities to relate knowledge and skills to wider contexts will motivate students to learn and to become lifelong learners.

Principals. The principal works in partnership with teachers and parents to ensure that each student has access to the best possible educational experience. To support student learning, principals ensure that the Ontario curriculum is being properly implemented in all classrooms through the use of a variety of instructional approaches, and that appropriate resources are made available for teachers and students. To enhance teaching and student learning in all subjects, including mathematics, principals promote learning teams and work with teachers to facilitate teacher participation in professional development activities. Principals are also responsible for ensuring that every student who has an Individual Education Plan (IEP) is receiving the modifications and/or accommodations described in his or her plan – in other words, for ensuring that the IEP is properly developed, implemented, and monitored.

The Program in Mathematics

Curriculum Expectations

The Ontario Curriculum, Grades 1 to 8: Mathematics, 2005 identifies the expectations for each grade and describes the knowledge and skills that students are expected to acquire, demonstrate, and apply in their class work and investigations, on tests, and in various other activities on which their achievement is assessed and evaluated.

Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

- The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.
- The *specific expectations* describe the expected knowledge and skills in greater detail. The specific expectations are grouped under subheadings that reflect particular aspects of the required knowledge and skills and that may serve as a guide for teachers as they plan learning activities for their students. (These groupings often reflect the "big ideas" of mathematics that are addressed in the strand.) The organization of expectations in subgroups is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups. The subheadings are used merely to help teachers focus on particular aspects of knowledge and skills as they develop and present various lessons and learning activities for their students.

In addition to the expectations outlined within each strand, a list of seven "mathematical process expectations" precedes the strands in each grade. These specific expectations describe the key processes essential to the effective study of mathematics, which students need to learn and apply throughout the year, regardless of the strand being studied. Teachers should ensure that students develop their ability to apply these processes in appropriate ways as they work towards meeting the expectations outlined in all the strands.

When developing their mathematics program and units of study from this document, teachers are expected to weave together related expectations from different strands, as well as the relevant mathematical process expectations, in order to create an overall program that integrates and balances concept development, skill acquisition, the use of processes, and applications.

Many of the expectations are accompanied by examples and/or sample problems, given in parentheses. These examples and sample problems are meant to illustrate the specific area of learning, the kind of skill, the depth of learning, and/or the level of complexity that the expectation entails. The examples are intended as a guide for teachers rather than as an exhaustive or mandatory list. Teachers do not have to address the full list of examples; rather, they may select one or two examples from the list and focus also on closely related areas of their own choosing. Similarly, teachers are not required to use the sample problems supplied. They may incorporate the sample problems into their lessons, or they may use other problems that are relevant to the expectation. Teachers will notice that some of the sample problems not only address the requirements of the expectation at hand but also incorporate knowledge or skills described in expectations in other strands of the same grade.

Some of the examples provided appear in quotation marks. These are examples of "student talk", and are offered to provide further clarification of what is expected of students. They illustrate how students might articulate observations or explain results related to the knowledge and skills outlined in the expectation. These examples are included to emphasize the importance of encouraging students to talk about the mathematics they are doing, as well as to provide some guidance for teachers in how to model mathematical language and reasoning for their students. As a result, they may not always reflect the exact level of language used by students in the particular grade.

Strands in the Mathematics Curriculum

Overall and specific expectations in mathematics are organized into five strands, which are the five major areas of knowledge and skills in the mathematics curriculum.

The program in all grades is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways. To support this process, teachers will, whenever possible, integrate concepts from across the five strands and apply the mathematics to real-life situations.

The five strands are Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability.

Number Sense and Numeration. Number sense refers to a general understanding of number and operations as well as the ability to apply this understanding in flexible ways to make mathematical judgements and to develop useful strategies for solving problems. In this strand, students develop their understanding of number by learning about different ways of representing numbers and about the relationships among numbers. They learn how to count in various ways, developing a sense of magnitude. They also develop a solid understanding of the four basic operations and learn to compute fluently, using a variety of tools and strategies.

A well-developed understanding of number includes a grasp of more-and-less relationships, part-whole relationships, the role of special numbers such as five and ten, connections between numbers and real quantities and measures in the environment, and much more.

Experience suggests that students do not grasp all of these relationships automatically. A broad range of activities and investigations, along with guidance by the teacher, will help students construct an understanding of number that allows them to make sense of mathematics and to know how and when to apply relevant concepts, strategies, and operations as they solve problems.

Measurement. Measurement concepts and skills are directly applicable to the world in which students live. Many of these concepts are also developed in other subject areas, such as science, social studies, and physical education.

In this strand, students learn about the measurable attributes of objects and about the units and processes involved in measurement. Students begin to learn how to measure by working with non-standard units, and then progress to using the basic metric units to measure quantities such as length, area, volume, capacity, mass, and temperature. They identify benchmarks to help them recognize the magnitude of units such as the kilogram, the litre, and the metre. Skills associated with telling time and computing elapsed time are also developed. Students learn about important relationships among measurement units and about relationships involved in calculating the perimeters, areas, and volumes of a variety of shapes and figures.

Concrete experience in solving measurement problems gives students the foundation necessary for using measurement tools and applying their understanding of measurement relationships. Estimation activities help students to gain an awareness of the size of different units and to become familiar with the process of measuring. As students' skills in numeration develop, they can be challenged to undertake increasingly complex measurement problems, thereby strengthening their facility in both areas of mathematics.

Geometry and Spatial Sense. Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Geometry helps us represent and describe objects and their interrelationships in space. A strong sense of spatial relationships and competence in using the concepts and language of geometry also support students' understanding of number and measurement.

Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Insights and intuitions about the characteristics of two-dimensional shapes and three-dimensional figures, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions.

In this strand, students learn to recognize basic shapes and figures, to distinguish between the attributes of an object that are geometric properties and those that are not, and to investigate the shared properties of classes of shapes and figures. Mathematical concepts and skills related to location and movement are also addressed in this strand.

Patterning and Algebra. One of the central themes in mathematics is the study of patterns and relationships. This study requires students to recognize, describe, and generalize patterns and to build mathematical models to simulate the behaviour of real-world phenomena that exhibit observable patterns.

Young students identify patterns in shapes, designs, and movement, as well as in sets of numbers. They study both repeating patterns and growing and shrinking patterns and develop ways to extend them. Concrete materials and pictorial displays help students create patterns and recognize relationships. Through the observation of different representations of a pattern, students begin to identify some of the properties of the pattern.

In the junior grades, students use graphs, tables, and verbal descriptions to represent relationships that generate patterns. Through activities and investigations, students examine how patterns change, in order to develop an understanding of variables as changing quantities. In the intermediate grades, students represent patterns algebraically and use these representations to make predictions.

A second focus of this strand is on the concept of equality. Students look at different ways of using numbers to represent equal quantities. Variables are introduced as "unknowns", and techniques for solving equations are developed. Problem solving provides students with opportunities to develop their ability to make generalizations and to deepen their understanding of the relationship between patterning and algebra.

Data Management and Probability. The related topics of data management and probability are highly relevant to everyday life. Graphs and statistics bombard the public in advertising, opinion polls, population trends, reliability estimates, descriptions of discoveries by scientists, and estimates of health risks, to name just a few.

In this strand, students learn about different ways to gather, organize, and display data. They learn about different types of data and develop techniques for analysing the data that include determining measures of central tendency and examining the distribution of the data. Students also actively explore probability by conducting probability experiments and using probability models to simulate situations. The topic of probability offers a wealth of interesting problems that can fascinate students and that provide a bridge to other topics, such as ratios, fractions, percents, and decimals. Connecting probability and data management to real-world problems helps make the learning relevant to students.

The Mathematical Processes

Presented at the start of every grade outlined in this curriculum document is a set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations outlined within the five strands. The need to highlight these process expectations arose from the recognition that students should be actively engaged in applying these processes throughout the program, rather than in connection with particular strands.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The mathematical processes can be seen as the processes through which students acquire and apply mathematical knowledge and skills. These processes are interconnected. Problem solving and communicating have strong links to all the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to make conjectures and justify solutions, orally and in writing. The communication and reflection that occur during and after the process of problem solving help students not only to articulate and refine their thinking but also to see the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to reflect on their own thinking (a process known as "metacognition") and the thinking of others, and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the year. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in all the strands in every grade.

Problem Solving

Problem solving is central to learning mathematics. By learning to solve problems and by learning *through* problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction.

It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident in their ability to do mathematics;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (estimating, evaluating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, and making judgements).

Not all mathematics instruction, however, can take place in a problem-solving context. Certain aspects of mathematics need to be taught explicitly. Mathematical conventions, including the use of mathematical symbols and terms, are one such aspect, and they should be introduced to students as needed, to enable them to use the symbolic language of mathematics.

Selecting Problem-Solving Strategies. Problem-solving strategies are methods that can be used to solve problems of various types. Teachers who use relevant and meaningful problem-solving experiences as the focus of their mathematics class help students to develop and extend a repertoire of strategies and methods that they can apply when solving various kinds of problems – instructional problems, routine problems, and non-routine problems. Students develop this repertoire over time, as they become more mature in their problem-solving skills. Eventually, students will have learned many problem-solving strategies that they can flexibly use and integrate when faced with new problem-solving situations, or to learn or reinforce mathematical concepts. Common problem-solving strategies include the following: making a model, picture, or diagram; looking for a pattern; guessing and checking; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

The Four-Step Problem-Solving Model. Students who have a good understanding of mathematical concepts may still have difficulty applying their knowledge in problem-solving activities, because they have not yet internalized a model that can guide them through the process. The most commonly used problem-solving model is George Polya's four-step model: understand the problem; make a plan; carry out the plan; and look back to check the results.[1] (These four steps are now reflected in the Thinking category of the achievement chart.)

1. First published in Polya's *How to Solve It*, 1945.

The four-step model is generally not taught directly before Grade 3, because young students tend to become too focused on the model and pay less attention to the mathematical concepts involved and to making sense of the problem. However, a teacher who is aware of the model and who uses it to guide his or her questioning and prompting during the problem-solving process will help students internalize a valuable approach that can be generalized to other problem-solving situations, not only in mathematics but in other subjects as well.

The four-step model provides a framework for helping students to think about a question before, during, and after the problem-solving experience. By Grade 3, the teacher can present the problem-solving model more explicitly, building on students' experiences in the earlier grades. The four-step model can then be displayed in the classroom and referred to often during the mathematics lesson.

The stages of the four-step model are described in Figure 1. Students should be made aware that, although the four steps are presented sequentially, it may sometimes be necessary in the course of solving problems to go back and revisit earlier steps.

Figure 1: A Problem-Solving Model

Understand the Problem (the exploratory stage)
➤ reread and restate the problem ➤ identify the information given and the information that needs to be determined ***Communication:*** talk about the problem to understand it better
Make a Plan
➤ relate the problem to similar problems solved in the past ➤ consider possible strategies ➤ select a strategy or a combination of strategies ***Communication:*** discuss ideas with others to clarify which strategy or strategies would work best
Carry Out the Plan
➤ execute the chosen strategy ➤ do the necessary calculations ➤ monitor success ➤ revise or apply different strategies as necessary ***Communication:*** ➤ draw pictures; use manipulatives to represent interim results ➤ use words and symbols to represent the steps in carrying out the plan or doing the calculations ➤ share results of computer or calculator operations
Look Back at the Solution
➤ check the reasonableness of the answer ➤ review the method used: Did it make sense? Is there a better way to approach the problem? ➤ consider extensions or variations ***Communication:*** describe how the solution was reached, using the most suitable format, and explain the solution

Reasoning and Proving

The reasoning process supports a deeper understanding of mathematics by enabling students to make sense of the mathematics they are learning. The process involves exploring phenomena, developing ideas, making mathematical conjectures, and justifying results. Teachers draw on students' natural ability to reason to help them learn to reason mathematically. Initially, students may rely on the viewpoints of others to justify a choice or an approach. Students should be encouraged to reason from the evidence they find in their explorations and investigations or from what they already know to be true, and to recognize the characteristics of an acceptable argument in the mathematics classroom. Teachers help students revisit conjectures that they have found to be true in one context to see if they are always true. For example, when teaching students in the junior grades about decimals, teachers may guide students to revisit the conjecture that multiplication always makes things bigger.

Reflecting

Good problem solvers regularly and consciously reflect on and monitor their own thought processes. By doing so, they are able to recognize when the technique they are using is not fruitful, and to make a conscious decision to switch to a different strategy, rethink the problem, search for related content knowledge that may be helpful, and so forth. Students' problem-solving skills are enhanced when they reflect on alternative ways to perform a task, even if they have successfully completed it. Reflecting on the reasonableness of an answer by considering the original question or problem is another way in which students can improve their ability to make sense of problems. Even very young students should be taught to examine their own thought processes in this way.

One of the best opportunities for students to reflect is immediately after they have completed an investigation, when the teacher brings students together to share and analyse their solutions. Students then share strategies, defend the procedures they used, justify their answers, and clarify any misunderstandings they may have had. This is the time that students can reflect on what made the problem difficult or easy (e.g., there were too many details to consider; they were not familiar with the mathematical terms used) and think about how they might tackle the problem differently. Reflecting on their own thinking and the thinking of others helps students make important connections and internalize a deeper understanding of the mathematical concepts involved.

Selecting Tools and Computational Strategies

Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Calculators, Computers, Communications Technology. Various types of technology are useful in learning and doing mathematics. Although students must develop basic operational skills, calculators and computers can help them extend their capacity to investigate and analyse mathematical concepts and reduce the time they might otherwise spend on purely mechanical activities.

Students can use calculators or computers to perform operations, make graphs, and organize and display data that are lengthier and more complex than those that might be addressed using only pencil-and-paper. Students can also use calculators and computers in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving. When students use calculators and computers in mathematics, they need to know when it is appropriate to apply their mental computation, reasoning, and estimation skills to predict and check answers.

The computer and the calculator should be seen as important problem-solving tools to be used for many purposes. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the particular applications that may be helpful to them as they search for their own solutions to problems.

Students may not be familiar with the use of some of the technologies suggested in the curriculum. When this is the case, it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated. Students also need to understand the situations in which the new technology would be an appropriate choice of tool. Students' use of the tools should not be laborious or restricted to inputting or following a set of procedures. For example, when using spreadsheets and dynamic statistical software (e.g., *TinkerPlots*), teachers could supply students with prepared data sets, and when using dynamic geometry software (e.g., *The Geometer's Sketchpad*), they could use pre-made sketches so that students' work with the software would be focused on the mathematics related to the data or on the manipulation of the sketch, not on the inputting of data or the designing of the sketch.

Computer programs can help students to collect, organize, and sort the data they gather, and to write, edit, and present reports on their findings. Whenever appropriate, students should be encouraged to select and use the communications technology that would best support and communicate their learning. Students, working individually or in groups, can use computers, CD-ROM technology, and/or Internet websites to gain access to Statistics Canada, mathematics organizations, and other valuable sources of mathematical information around the world.

Manipulatives.[2] Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others. Using manipulatives to construct representations helps students to:

- see patterns and relationships;
- make connections between the concrete and the abstract;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others.

Computational Strategies. Problem solving often requires students to select an appropriate computational strategy. They may need to apply the written procedures (or algorithms) for addition, subtraction, multiplication, or division or use technology for computation. They may also need to select strategies related to mental computation and estimation. Developing the ability to perform mental computations and to estimate is consequently an important aspect of student learning in mathematics.

2. See the Teaching Approaches section, on pages 24–26 of this document, for additional information about the use of manipulatives in mathematics instruction.

Mental computation involves calculations done in the mind, with little or no use of paper and pencil. Students who have developed the ability to calculate mentally can select from and use a variety of procedures that take advantage of their knowledge and understanding of numbers, the operations, and their properties. Using their knowledge of the distributive property, for example, students can mentally compute 70% of 22 by first considering 70% of 20 and then adding 70% of 2. Used effectively, mental computation can encourage students to think more deeply about numbers and number relationships.

Knowing how to estimate, and knowing when it is useful to estimate and when it is necessary to have an exact answer, are important mathematical skills. Estimation is a useful tool for judging the reasonableness of a solution and for guiding students in their use of calculators. The ability to estimate depends on a well-developed sense of number and an understanding of place value. It can be a complex skill that requires decomposing numbers, rounding, using compatible numbers, and perhaps even restructuring the problem. Estimation should not be taught as an isolated skill or a set of isolated rules and techniques. Knowing about calculations that are easy to perform and developing fluency in performing basic operations contribute to successful estimation.

Connecting

Experiences that allow students to make connections – to see, for example, how concepts and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps develop mathematical understanding. The more connections students make, the deeper their understanding. In addition, making connections between the mathematics they learn at school and its applications in their everyday lives not only helps students understand mathematics but also allows them to see how useful and relevant it is in the world beyond the classroom.

Representing

In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations.

Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. For example, a student in the primary grades should know how to represent four groups of two by means of repeated addition, counting by 2's, or using an array of objects. The array representation can help students begin to understand the commutative property (e.g., $2 \times 4 = 4 \times 2$), a concept that can help them simplify their computations. In the junior grades, students model and solve problems using representations that include pictures, tables, graphs, words, and symbols. Students in Grades 7 and 8 begin to use algebraic representations to model and interpret mathematical, physical, and social phenomena.

When students are able to represent concepts in various ways, they develop flexibility in their thinking about those concepts. They are not inclined to perceive any single representation as "the math"; rather, they understand that it is just one of many representations that help them understand a concept.

Communicating

Communication is the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and clarify their ideas, their understanding of mathematical relationships, and their mathematical arguments.

Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. For example, teachers can:

- model mathematical reasoning by thinking aloud, and encourage students to think aloud;
- model proper use of symbols, vocabulary, and notations in oral, visual, and written form;
- ensure that students begin to use new mathematical vocabulary as it is introduced (e.g., with the aid of a word wall; by providing opportunities to read, question, and discuss);
- provide feedback to students on their use of terminology and conventions;
- encourage talk at each stage of the problem-solving process;
- ask clarifying and extending questions and encourage students to ask themselves similar kinds of questions;
- ask students open-ended questions relating to specific topics or information (e.g., "How do you know?" "Why?" "What if …?", "What pattern do you see?", "Is this always true?");
- model ways in which various kinds of questions can be answered;
- encourage students to seek clarification when they are unsure or do not understand something.

Effective classroom communication requires a supportive and respectful environment that makes all members of the class feel comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher.

The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics.

Assessment and Evaluation of Student Achievement

Basic Considerations

The primary purpose of assessment and evaluation is to improve student learning. Information gathered through assessment helps teachers to determine students' strengths and weaknesses in their achievement of the curriculum expectations in each subject in each grade. This information also serves to guide teachers in adapting curriculum and instructional approaches to students' needs and in assessing the overall effectiveness of programs and classroom practices.

Assessment is the process of gathering information from a variety of sources (including assignments, day-to-day observations and conversations/conferences, demonstrations, projects, performances, and tests) that accurately reflects how well a student is achieving the curriculum expectations in a subject. As part of assessment, teachers provide students with descriptive feedback that guides their efforts towards improvement. Evaluation refers to the process of judging the quality of student work on the basis of established criteria, and assigning a value to represent that quality. In Ontario elementary schools, the value assigned will be in the form of a letter grade for Grades 1 to 6 and a percentage grade for Grades 7 and 8.

Assessment and evaluation will be based on the provincial curriculum expectations and the achievement levels outlined in this document.

In order to ensure that assessment and evaluation are valid and reliable, and that they lead to the improvement of student learning, teachers must use assessment and evaluation strategies that:

- address both what students learn and how well they learn;
- are based both on the categories of knowledge and skills and on the achievement level descriptions given in the achievement chart on pages 22–23;
- are varied in nature, administered over a period of time, and designed to provide opportunities for students to demonstrate the full range of their learning;
- are appropriate for the learning activities used, the purposes of instruction, and the needs and experiences of the students;
- are fair to all students;
- accommodate the needs of exceptional students, consistent with the strategies outlined in their Individual Education Plan;
- accommodate the needs of students who are learning the language of instruction (English or French);
- ensure that each student is given clear directions for improvement;
- promote students' ability to assess their own learning and to set specific goals;
- include the use of samples of students' work that provide evidence of their achievement;
- are communicated clearly to students and parents at the beginning of the school year and at other appropriate points throughout the year.

All curriculum expectations must be accounted for in instruction, but evaluation focuses on students' achievement of the overall expectations. A student's achievement of the overall expectations is evaluated on the basis of his or her achievement of related specific expectations (including the mathematical process expectations). The overall expectations are broad in nature, and the specific expectations define the particular content or scope of the knowledge and skills referred to in the overall expectations. Teachers will use their professional judgement to determine which specific expectations should be used to evaluate achievement of the overall expectations, and which ones will be covered in instruction and assessment (e.g., through direct observation) but not necessarily evaluated.

The characteristics given in the achievement chart (pages 22–23) for level 3 represent the "provincial standard" for achievement of the expectations. A complete picture of overall achievement at level 3 in mathematics can be constructed by reading from top to bottom in the shaded column of the achievement chart, headed "Level 3". Parents of students achieving at level 3 can be confident that their children will be prepared for work in the next grade.

Level 1 identifies achievement that falls much below the provincial standard, while still reflecting a passing grade. Level 2 identifies achievement that approaches the standard. Level 4 identifies achievement that surpasses the standard. It should be noted that achievement at level 4 does not mean that the student has achieved expectations beyond those specified for a particular grade. It indicates that the student has achieved all or almost all of the expectations for that grade, and that he or she demonstrates the ability to use the knowledge and skills specified for that grade in more sophisticated ways than a student achieving at level 3.

The Ministry of Education provides teachers with materials that will assist them in improving their assessment methods and strategies and, hence, their assessment of student achievement. These materials include samples of student work (exemplars) that illustrate achievement at each of the four levels.

The Achievement Chart for Mathematics

The achievement chart that follows identifies four categories of knowledge and skills in mathematics. The achievement chart is a standard province-wide guide to be used by teachers. It enables teachers to make judgements about student work that are based on clear performance standards and on a body of evidence collected over time.

The purpose of the achievement chart is to:

- provide a framework that encompasses all curriculum expectations for all grades and subjects represented in this document;
- guide the development of assessment tasks and tools (including rubrics);
- help teachers to plan instruction for learning;
- assist teachers in providing meaningful feedback to students;
- provide various categories and criteria with which to assess and evaluate student learning.

Categories of knowledge and skills. The categories, defined by clear criteria, represent four broad areas of knowledge and skills within which the subject expectations for any given grade are organized. The four categories should be considered as interrelated, reflecting the wholeness and interconnectedness of learning.

The categories of knowledge and skills are described as follows:

Knowledge and Understanding. Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding).

Thinking. The use of critical and creative thinking skills and/or processes,[3] as follows:
– planning skills (e.g., understanding the problem, making a plan for solving the problem)
– processing skills (e.g., carrying out a plan, looking back at the solution)
– critical/creative thinking processes (e.g., inquiry, problem solving)

Communication. The conveying of meaning through various oral, written, and visual forms (e.g., providing explanations of reasoning or justification of results orally or in writing; communicating mathematical ideas and solutions in writing, using numbers and algebraic symbols, and visually, using pictures, diagrams, charts, tables, graphs, and concrete materials).

Application. The use of knowledge and skills to make connections within and between various contexts.

Teachers will ensure that student work is assessed and/or evaluated in a balanced manner with respect to the four categories, and that achievement of particular expectations is considered within the appropriate categories.

Criteria. Within each category in the achievement chart, criteria are provided, which are subsets of the knowledge and skills that define each category. For example, in Knowledge and Understanding, the criteria are "knowledge of content (e.g., facts, terms, procedural skills, use of tools)" and "understanding of mathematical concepts". The criteria identify the aspects of student performance that are assessed and/or evaluated, and serve as guides to what to look for.

Descriptors. A "descriptor" indicates the characteristic of the student's performance, with respect to a particular criterion, on which assessment or evaluation is focused. In the achievement chart, *effectiveness* is the descriptor used for each criterion in the Thinking, Communication, and Application categories. What constitutes effectiveness in any given performance task will vary with the particular criterion being considered. Assessment of effectiveness may therefore focus on a quality such as appropriateness, clarity, accuracy, precision, logic, relevance, significance, fluency, flexibility, depth, or breadth, as appropriate for the particular criterion. For example, in the Thinking category, assessment of effectiveness might focus on the degree of relevance or depth apparent in an analysis; in the Communication category, on clarity of expression or logical organization of information and ideas; or in the Application category, on appropriateness or breadth in the making of connections. Similarly, in the Knowledge and Understanding category, assessment of knowledge might focus on accuracy, and assessment of understanding might focus on the depth of an explanation. Descriptors help teachers to focus their assessment and evaluation on specific knowledge and skills for each category and criterion, and help students to better understand exactly what is being assessed and evaluated.

Qualifiers. A specific "qualifier" is used to define each of the four levels of achievement – that is, *limited* for level 1, *some* for level 2, *considerable* for level 3, and *a high degree* or *thorough* for level 4. A qualifier is used along with a descriptor to produce a description of performance at a particular level. For example, the description of a student's performance at level 3 with respect to the first criterion in the Thinking category would be: "The student uses planning skills with *considerable* effectiveness".

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3. See the footnote on page 22, pertaining to the *mathematical processes*.

The descriptions of the levels of achievement given in the chart should be used to identify the level at which the student has achieved the expectations. Students should be provided with numerous and varied opportunities to demonstrate the full extent of their achievement of the curriculum expectations, across all four categories of knowledge and skills.

Achievement Chart – Mathematics, Grades 1–8

Categories	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding *Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)*				
	The student:			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
Thinking *The use of critical and creative thinking skills and/or processes**				
	The student:			
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills* – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes* (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

* The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* described on pages 11–17 of this document. Some aspects of the mathematical processes relate to the other categories of the achievement chart.

Categories	Level 1	Level 2	Level 3	Level 4
Communication *The conveying of meaning through various forms*				
	The student:			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application *The use of knowledge and skills to make connections within and between various contexts*				
	The student:			
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	– applies knowledge and skills in familiar contexts with considerable effectiveness	– applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	– transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

Some Considerations for Program Planning in Mathematics

When planning a program in mathematics, teachers must take into account considerations in a number of important areas, including those discussed below.

The Ministry of Education has produced or supported the production of a variety of resource documents that teachers may find helpful as they plan programs based on the expectations outlined in this curriculum document. They include the following:

- *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2005* (forthcoming; replaces the 2003 edition for Kindergarten to Grade 3), along with companion documents focusing on individual strands
- *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario, 2003*
- *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004*
- *Leading Math Success: Mathematical Literacy, Grades 7–12 – The Report of the Expert Panel on Student Success in Ontario, 2004*
- *Think Literacy: Cross-Curricular Approaches, Grades 7–12 – Mathematics: Subject-Specific Examples, Grades 7–9, 2004*
- *Targeted Implementation & Planning Supports (TIPS): Grades 7, 8, and 9 Applied Mathematics, 2003*

Teaching Approaches

Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students.

In order to learn mathematics and to apply their knowledge effectively, students must develop a solid understanding of mathematical concepts. Research and successful classroom practice have shown that an investigative approach, with an emphasis on learning through problem solving and reasoning, best enables students to develop the conceptual foundation they need. When planning mathematics programs, teachers will provide activities and assignments that encourage students to search for patterns and relationships and engage in logical inquiry. Teachers need to use rich problems and present situations that provide a variety of opportunities for students to develop mathematical understanding through problem solving.

All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas", or key principles, of mathematics, such as pattern or relationship. This understanding of key principles will enable and encourage students to use mathematical reasoning throughout their lives.

Effective instructional approaches and learning activities draw on students' prior knowledge, capture their interest, and encourage meaningful practice both inside and outside the classroom. Students' interest will be engaged when they are able to see the connections between the mathematical concepts they are learning and their application in the world around them and in real-life situations.

Students will investigate mathematical concepts using a variety of tools and strategies, both manual and technological. Manipulatives are necessary tools for supporting the effective learning of mathematics by all students. These concrete learning tools invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Moreover, using a variety of manipulatives helps deepen and extend students' understanding of mathematical concepts. For example, students who have used only base ten materials to represent two-digit numbers may not have as strong a conceptual understanding of place value as students who have also bundled craft sticks into tens and hundreds and used an abacus.

Manipulatives are also a valuable aid to teachers. By analysing students' concrete representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance their thinking.[4]

Fostering students' communication skills is an important part of the teacher's role in the mathematics classroom. Through skilfully led classroom discussions, students build understanding and consolidate their learning. Discussions provide students with the opportunity to ask questions, make conjectures, share and clarify ideas, suggest and compare strategies, and explain their reasoning. As they discuss ideas with their peers, students learn to discriminate between effective and ineffective strategies for problem solving.

Students' understanding is revealed through both oral communication and writing, but it is not necessary for all mathematics learning to involve a written communication component. Young students need opportunities to focus on their oral communication without the additional responsibility of writing.

Whether students are talking or writing about their mathematical learning, teachers can prompt them to explain their thinking and the mathematical reasoning behind a solution or the use of a particular strategy by asking the question "How do you know?". And because mathematical reasoning must be the primary focus of students' communication, it is important for teachers to select instructional strategies that elicit mathematical reasoning from their students.

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4. Lists of manipulatives appropriate for use in elementary classrooms are provided in the expert panel reports on mathematics, as follows: *Early Math Strategy: The Report of the Expert Panel on Early Mathematics in Ontario, 2003*, pp 21–24; *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004*, pp. 61–63; *Leading Math Success: Mathematical Literacy, Grades 7–12 – The Report of the Expert Panel on Student Success in Ontario, 2004*, pp. 48–49.

Promoting Positive Attitudes Towards Mathematics. Students' attitudes have a significant effect on how they approach problem solving and how well they succeed in mathematics. Teachers can help students develop the confidence they need by demonstrating a positive disposition towards mathematics.[5] Students need to understand that, for some mathematics problems, there may be several ways to arrive at the correct answer. They also need to believe that they are capable of finding solutions. It is common for people to think that if they cannot solve problems quickly and easily, they must be inadequate. Teachers can help students understand that problem solving of almost any kind often requires a considerable expenditure of time and energy and a good deal of perseverance. Once students have this understanding, teachers can encourage them to develop the willingness to persist, to investigate, to reason and explore alternative solutions, and to take the risks necessary to become successful problem solvers.

Cross-Curricular and Integrated Learning

The development of skills and knowledge in mathematics is often enhanced by learning in other subject areas. Teachers should ensure that all students have ample opportunities to explore a subject from multiple perspectives by emphasizing cross-curricular learning and integrated learning, as follows:

a) In cross-curricular learning, students are provided with opportunities to learn and use related content and/or skills in two or more subjects. Students can use the concepts and skills of mathematics in their science or social studies lessons. Similarly, students can use what they have learned in science to illustrate or develop mathematical understanding. For example, in Grade 6, concepts associated with the fulcrum of a lever can be used to develop a better understanding of the impact that changing a set of data can have on the mean.

b) In integrated learning, students are provided with opportunities to work towards *meeting expectations from two or more subjects* within a single unit, lesson, or activity. By linking expectations from different subject areas, teachers can provide students with multiple opportunities to reinforce and demonstrate their knowledge and skills in a range of settings. Also, the mathematical process expectation that focuses on connecting encourages students to make connections between mathematics and other subject areas. For example, students in Grade 2 could be given the opportunity to relate the study of location and movement in the Geometry and Spatial Sense strand of mathematics to the study of movement in the Structures and Mechanisms strand in science and technology. Similarly, the same students could link their study of the characteristics of symmetrical shapes in Visual Arts to the creation of symmetrical shapes in their work in Geometry and Spatial Sense.

Planning Mathematics Programs for Exceptional Students

In planning mathematics programs for exceptional students, teachers should begin by examining both the curriculum expectations for the appropriate grade level and the needs of the individual student to determine which of the following options is appropriate for the student:

- no accommodations[6] or modifications; or
- accommodations only; or
- modified expectations, with the possibility of accommodations.

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5. *Leading Math Success*, p. 42.
6. "Accommodations" refers to individualized teaching and assessment strategies, human supports, and/or individualized equipment.

If the student requires either accommodations or modified expectations, or both, the relevant information, as described in the following paragraphs, must be recorded in his or her Individual Education Plan (IEP). For a detailed discussion of the ministry's requirements for IEPs, see *Individual Education Plans: Standards for Development, Program Planning, and Implementation, 2000* (referred to hereafter as *IEP Standards, 2000*). More detailed information about planning programs for exceptional students can be found in *The Individual Education Plan (IEP): A Resource Guide, 2004*. (Both documents are available at http://www.edu.gov.on.ca.)

Students Requiring Accommodations Only. With the aid of accommodations alone, some exceptional students are able to participate in the regular grade-level curriculum and to demonstrate learning independently. (Accommodations do not alter the provincial curriculum expectations for the grade level.)

The accommodations required to facilitate the student's learning must be identified in his or her IEP (see *IEP Standards, 2000*, page 11). A student's IEP is likely to reflect the same accommodations for many, or all, subject areas.

There are three types of accommodations. *Instructional accommodations* are changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia. *Environmental accommodations* are changes that the student may require in the classroom and/or school environment, such as preferential seating or special lighting. *Assessment accommodations* are changes in assessment procedures that enable the student to demonstrate his or her learning, such as allowing additional time to complete tests or assignments or permitting oral responses to test questions (see page 29 of *The Individual Education Plan (IEP): A Resource Guide, 2004*, for more examples).

If a student requires "accommodations only" in mathematics, assessment and evaluation of his or her achievement will be based on the appropriate grade-level curriculum expectations and the achievement levels outlined in this document.

Students Requiring Modified Expectations. Some exceptional students will require modified expectations, which differ from the regular grade-level expectations. In mathematics, modified expectations will usually be based on the knowledge and skills outlined in curriculum expectations for a different grade level. Modified expectations must indicate the knowledge and/or skills the student is expected to demonstrate and have assessed in each reporting period (*IEP Standards, 2000*, pages 10 and 11). Students requiring modified expectations need to develop knowledge and skills in all five strands of the mathematics curriculum. Modified expectations must represent specific, realistic, observable, and measurable achievements and must describe specific knowledge and/or skills that the student can demonstrate independently, given the appropriate assessment accommodations. They should be expressed in such a way that the student and parents can understand exactly what the student is expected to know or be able to do, on the basis of which his or her performance will be evaluated and a grade or mark recorded on the Provincial Report Card. The grade level of the learning expectations must be identified in the student's IEP. The student's learning expectations must be reviewed in relation to the student's progress at least once every reporting period, and must be updated as necessary (*IEP Standards, 2000*, page 11).

If a student requires modified expectations in mathematics, assessment and evaluation of his or her achievement will be based on the learning expectations identified in the IEP and on the achievement levels outlined in this document. On the Provincial Report Card, the IEP box must be checked for any subject in which the student requires modified expectations, and the appropriate statement from the *Guide to the Provincial Report Card, Grades 1–8, 1998* (page 8) must be inserted. The teacher's comments should include relevant information on the student's demonstrated learning of the modified expectations, as well as next steps for the student's learning in the subject.

English As a Second Language and English Literacy Development (ESL/ELD)

Young people whose first language is not English enter Ontario elementary schools with diverse linguistic and cultural backgrounds. Some may have experience of highly sophisticated educational systems while others may have had limited formal schooling. All of these students bring a rich array of background knowledge and experience to the classroom, and all teachers must share in the responsibility for their English-language development.

Teachers of mathematics need to incorporate appropriate instructional and assessment strategies to help ESL and ELD students succeed in their classrooms. These strategies include:

- modification of some or all of the curriculum expectations, based on the student's level of English proficiency;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, manipulatives, pictures, diagrams, graphic organizers; attention to the clarity of instructions; modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key specialized vocabulary; encouragement of peer tutoring and class discussion; strategic use of students' first languages);
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, culturally diverse materials);
- use of assessment accommodations (e.g., granting of extra time; use of alternative forms of assessment, such as oral interviews, learning logs, or portfolios; simplification of language used in problems and instructions).

See *The Ontario Curriculum, Grades 1–8: English As a Second Language and English Literacy Development – A Resource Guide, 2001* (available at www.edu.gov.on.ca) for detailed information about modifying expectations for ESL/ELD students and about assessing, evaluating, and reporting on student achievement.

Antidiscrimination Education in Mathematics

To ensure that all students in the province have an equal opportunity to achieve their full potential, the curriculum must be free from bias and all students must be provided with a safe and secure environment, characterized by respect for others, that allows them to participate fully and responsibly in the educational experience.

Learning activities and resources used to implement the curriculum should be inclusive in nature, reflecting the range of experiences of students with varying backgrounds, abilities, interests, and learning styles. They should enable students to become more sensitive to the diverse cultures and perceptions of others, including Aboriginal peoples. For example, activities can be designed to relate concepts in geometry or patterning to the arches and tile work often found in Asian architecture or to the patterns used in Aboriginal basketry design. By discussing aspects of the history of mathematics, teachers can help make students aware of the various cultural groups that have contributed to the evolution of mathematics over the centuries. Finally, students need to recognize that ordinary people use mathematics in a variety of everyday contexts, both at work and in their daily lives.

Connecting mathematical ideas to real-world situations through learning activities can enhance students' appreciation of the role of mathematics in human affairs, in areas including health, science, and the environment. Students can be made aware of the use of mathematics in contexts such as sampling and surveying and the use of statistics to analyse trends. Recognizing the importance of mathematics in such areas helps motivate students to learn and also provides a foundation for informed, responsible citizenship.

Teachers should have high expectations for all students. To achieve their mathematical potential, however, different students may need different kinds of support. Some boys, for example, may need additional support in developing their literacy skills in order to complete mathematical tasks effectively. For some girls, additional encouragement to envision themselves in careers involving mathematics may be beneficial. For example, teachers might consider providing strong role models in the form of female guest speakers who are mathematicians or who use mathematics in their careers.

Literacy and Inquiry/Research Skills

Literacy skills can play an important role in student success in mathematics. Many of the activities and tasks students undertake in mathematics involve the use of written, oral, and visual communication skills. For example, students use language to record their observations, to explain their reasoning when solving problems, to describe their inquiries in both informal and formal contexts, and to justify their results in small-group conversations, oral presentations, and written reports. The language of mathematics includes special terminology. The study of mathematics consequently encourages students to use language with greater care and precision and enhances their ability to communicate effectively.

Some of the literacy strategies that can be helpful to students as they learn mathematics include the following: reading strategies that help students build vocabulary and improve their ability to navigate textbooks; writing strategies that help students sort ideas and information in order to make connections, identify relationships, and determine possible directions for their thinking and writing; and oral communication strategies that help students communicate in small-group and whole-class discussions. Further advice for integrating literacy instruction into mathematics instruction in the intermediate grades may be found in the following resource documents:

- *Think Literacy: Cross-Curricular Approaches, Grades 7–12, 2003*
- *Think Literacy: Cross-Curricular Approaches, Grades 7–12 – Mathematics: Subject-Specific Examples, Grades 7–9, 2004*

As they solve problems, students develop their ability to ask questions and to plan investigations to answer those questions and to solve related problems. In their work in the Data Management and Probability strand, students learn to apply a variety of inquiry and research methods as they solve statistical problems. Students also learn how to locate relevant information from a variety of sources such as statistical databases, newspapers, and reports.

The Role of Technology in Mathematics

Information and communication technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning in mathematics. Teachers can use ICT tools and resources both for whole class instruction and to design programs that meet diverse student needs. Technology can help to reduce the time spent on routine mathematical tasks and to promote thinking and concept development.

Technology can influence both what is taught in mathematics courses and how it is taught. Powerful assistive and enabling computer and handheld technologies can be used seamlessly in teaching, learning, and assessment. These tools include simulations, multimedia resources, databases, access to large amounts of statistical data, and computer-assisted learning modules.

Information and communications technologies can also be used in the classroom to connect students to other schools, at home and abroad, and to bring the global community into the local classroom.

Guidance and Mathematics

The guidance and career education program should be aligned with the mathematics curriculum. Teachers need to ensure that the classroom learning across all grades and subjects provides ample opportunity for students to learn how to work independently (e.g., complete homework independently), cooperate with others, resolve conflicts, participate in class, solve problems, and set goals to improve their work.

Teachers can help students to think of mathematics as a career option by pointing out the role of mathematics in the careers of people whom students observe in the community or in careers that students might be considering with their own future in mind. The mathematics program can also offer career exploration activities that include visits from guest speakers, contacts with career mentors, involvement in simulation programs (e.g., Junior Achievement programs), and attendance at career conferences.

Health and Safety in Mathematics

Although health and safety issues are not usually associated with mathematics, they may be important when investigation involves fieldwork. Out-of-school fieldwork, for example measuring playground or park dimensions, can provide an exciting and authentic dimension to students' learning experiences. Teachers must preview and plan these activities carefully to protect students' health and safety.

Grade 1

The following are highlights of student learning in Grade 1. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering whole numbers to 50; establishing the conservation of number; representing money amounts to 20¢; decomposing and composing numbers to 20; establishing a one-to-one correspondence when counting the elements in a set; counting by 1's, 2's, 5's, and 10's; adding and subtracting numbers to 20

Measurement: measuring using non-standard units; telling time to the nearest half-hour; developing a sense of area; comparing objects using measurable attributes; comparing objects using non-standard units; investigating the relationship between the size of a unit and the number of units needed to measure the length of an object

Geometry and Spatial Sense: sorting and classifying two-dimensional shapes and three-dimensional figures by attributes; recognizing symmetry; relating shapes to other shapes, to designs, and to figures; describing location using positional language

Patterning and Algebra: creating and extending repeating patterns involving one attribute; introducing the concept of equality using only concrete materials

Data Management and Probability: organizing objects into categories using one attribute; collecting and organizing categorical data; reading and displaying data using concrete graphs and pictographs; describing the likelihood that an event will occur

Grade 1: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 1, students will:

PROBLEM SOLVING
- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;

REPRESENTING
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

Grade 1: Number Sense and Numeration

Overall Expectations

By the end of Grade 1, students will:

- read, represent, compare, and order whole numbers to 50, and use concrete materials to investigate fractions and money amounts;
- demonstrate an understanding of magnitude by counting forward to 100 and backwards from 20;
- solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of strategies.

Specific Expectations

Quantity Relationships

By the end of Grade 1, students will:

– represent, compare, and order whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences, number stories);

– read and print in words whole numbers to ten, using meaningful contexts (e.g., storybooks, posters);

– demonstrate, using concrete materials, the concept of conservation of number (e.g., 5 counters represent the number 5, regardless whether they are close together or far apart);

– relate numbers to the anchors of 5 and 10 (e.g., 7 is 2 more than 5 and 3 less than 10);

– identify and describe various coins (i.e., penny, nickel, dime, quarter, $1 coin, $2 coin), using coin manipulatives or drawings, and state their value (e.g., the value of a penny is one cent; the value of a toonie is two dollars);

– represent money amounts to 20¢, through investigation using coin manipulatives;

– estimate the number of objects in a set, and check by counting (e.g., "I guessed that there were 20 cubes in the pile. I counted them and there were only 17 cubes. 17 is close to 20.");

– compose and decompose numbers up to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3);

– divide whole objects into parts and identify and describe, through investigation, equal-sized parts of the whole, using fractional names (e.g., halves; fourths or quarters).

Counting

By the end of Grade 1, students will:

– demonstrate, using concrete materials, the concept of one-to-one correspondence between number and objects when counting;

– count forward by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies (e.g., move with steps; skip count on a number line; place counters on a hundreds chart; connect cubes to show equal groups; count groups of pennies, nickels, or dimes);

– count backwards by 1's from 20 and any number less than 20 (e.g., count backwards from 18 to 11), with and without the use of concrete materials and number lines;
– count backwards from 20 by 2's and 5's, using a variety of tools (e.g., number lines, hundreds charts);
– use ordinal numbers to thirty-first in meaningful contexts (e.g., identify the days of the month on a calendar).

Operational Sense

By the end of Grade 1, students will:

– solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines) (*Sample problem:* Miguel has 12 cookies. Seven cookies are chocolate. Use counters to determine how many cookies are not chocolate.);
– solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles);
– add and subtract money amounts to 10¢, using coin manipulatives and drawings.

Grade 1: Measurement

Overall Expectations

By the end of Grade 1, students will:

- estimate, measure, and describe length, area, mass, capacity, time, and temperature, using non-standard units of the same size;
- compare, describe, and order objects, using attributes measured in non-standard units.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 1, students will:

– demonstrate an understanding of the use of non-standard units of the same size (e.g., straws, index cards) for measuring (*Sample problem:* Measure the length of your desk in different ways; for example, by using several different non-standard units or by starting measurements from opposite ends of the desk. Discuss your findings.);

– estimate, measure (i.e., by placing non-standard units repeatedly, without overlaps or gaps), and record lengths, heights, and distances (e.g., a book is about 10 paper clips wide; a pencil is about 3 toothpicks long);

– construct, using a variety of strategies, tools for measuring lengths, heights, and distances in non-standard units (e.g., footprints on cash register tape or on connecting cubes);

– estimate, measure (i.e., by minimizing overlaps and gaps), and describe area, through investigation using non-standard units (e.g., "It took about 15 index cards to cover my desk, with only a little bit of space left over.");

– estimate, measure, and describe the capacity and/or mass of an object, through investigation using non-standard units (e.g., "My journal has the same mass as 13 pencils." "The juice can has the same capacity as 4 pop cans.");

– estimate, measure, and describe the passage of time, through investigation using non-standard units (e.g., number of sleeps; number of claps; number of flips of a sand timer);

– read demonstration digital and analogue clocks, and use them to identify benchmark times (e.g., times for breakfast, lunch, dinner; the start and end of school; bedtime) and to tell and write time to the hour and half-hour in everyday settings;

– name the months of the year in order, and read the date on a calendar;

– relate temperature to experiences of the seasons (e.g., "In winter, we can skate because it's cold enough for there to be ice.").

Measurement Relationships

By the end of Grade 1, students will:

– compare two or three objects using measurable attributes (e.g., length, height, width, area, temperature, mass, capacity), and describe the objects using relative terms (e.g., *taller*, *heavier*, *faster*, *bigger*, *warmer*; "If I put an eraser, a pencil, and a metre stick beside each other, I can see that the eraser is shortest and the metre stick is longest.");

– compare and order objects by their linear measurements, using the same non-standard unit (*Sample problem:* Using a length of string equal to the length of your forearm, work with a partner to find other objects that are about the same length.);

- use the metre as a benchmark for measuring length, and compare the metre with non-standard units (*Sample problem:* In the classroom, use a metre stick to find objects that are taller than one metre and objects that are shorter than one metre.);
- describe, through investigation using concrete materials, the relationship between the size of a unit and the number of units needed to measure length (*Sample problem:* Compare the numbers of paper clips and pencils needed to measure the length of the same table.).

Grade 1: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 1, students will:

- identify common two-dimensional shapes and three-dimensional figures and sort and classify them by their attributes;*
- compose and decompose common two-dimensional shapes and three-dimensional figures;
- describe the relative locations of objects using positional language.

Specific Expectations

Geometric Properties

By the end of Grade 1, students will:

– identify and describe common two-dimensional shapes (e.g., circles, triangles, rectangles, squares) and sort and classify them by their attributes (e.g., colour; size; texture; number of sides), using concrete materials and pictorial representations (e.g., "I put all the triangles in one group. Some are long and skinny, and some are short and fat, but they all have three sides.");

– trace and identify the two-dimensional faces of three-dimensional figures, using concrete models (e.g., "I can see squares on the cube.");

– identify and describe common three-dimensional figures (e.g., cubes, cones, cylinders, spheres, rectangular prisms) and sort and classify them by their attributes (e.g., colour; size; texture; number and shape of faces), using concrete materials and pictorial representations (e.g., "I put the cones and the cylinders in the same group because they all have circles on them.");

– describe similarities and differences between an everyday object and a three-dimensional figure (e.g., "A water bottle looks like a cylinder, except the bottle gets thinner at the top.");

– locate shapes in the environment that have symmetry, and describe the symmetry.

Geometric Relationships

By the end of Grade 1, students will:

– compose patterns, pictures, and designs, using common two-dimensional shapes (*Sample problem:* Create a picture of a flower using pattern blocks.);

– identify and describe shapes within other shapes (e.g., shapes within a geometric design);

– build three-dimensional structures using concrete materials, and describe the two-dimensional shapes the structures contain;

– cover outline puzzles with two-dimensional shapes (e.g., pattern blocks, tangrams) (*Sample problem:* Fill in the outline of a boat with tangram pieces.).

Location and Movement

By the end of Grade 1, students will:

– describe the relative locations of objects or people using positional language (e.g., *over*, *under*, *above*, *below*, *in front of*, *behind*, *inside*, *outside*, *beside*, *between*, *along*);

* For the purposes of student learning in Grade 1, "attributes" refers to the various characteristics of two-dimensional shapes and three-dimensional figures, including geometric properties. (See glossary entries for "attribute" and "property (geometric)".) Students learn to distinguish attributes that are geometric properties from attributes that are not geometric properties in Grade 2.

– describe the relative locations of objects on concrete maps created in the classroom (*Sample problem:* Work with your group to create a map of the classroom in the sand table, using smaller objects to represent the classroom objects. Describe where the teacher's desk and the bookshelves are located.);
– create symmetrical designs and pictures, using concrete materials (e.g., pattern blocks, connecting cubes, paper for folding), and describe the relative locations of the parts.

Grade 1: Patterning and Algebra

Overall Expectations

By the end of Grade 1, students will:

- identify, describe, extend, and create repeating patterns;
- demonstrate an understanding of the concept of equality, using concrete materials and addition and subtraction to 10.

Specific Expectations

Patterns and Relationships

By the end of Grade 1, students will:

– identify, describe, and extend, through investigation, geometric repeating patterns involving one attribute (e.g., colour, size, shape, thickness, orientation);

– identify and extend, through investigation, numeric repeating patterns (e.g., 1, 2, 3, 1, 2, 3, 1, 2, 3, …);

– describe numeric repeating patterns in a hundreds chart;

– identify a rule for a repeating pattern (e.g., "We're lining up boy, girl, boy, girl, boy, girl.");

– create a repeating pattern involving one attribute (e.g., colour, size, shape, sound) (*Sample problem:* Use beads to make a string that shows a repeating pattern involving one attribute.);

– represent a given repeating pattern in a variety of ways (e.g., pictures, actions, colours, sounds, numbers, letters) (*Sample problem:* Make an ABA, ABA, ABA pattern using actions like clapping or tapping.).

Expressions and Equality

By the end of Grade 1, students will:

– create a set in which the number of objects is greater than, less than, or equal to the number of objects in a given set;

– demonstrate examples of equality, through investigation, using a "balance" model (*Sample problem:* Demonstrate, using a pan balance, that a train of 7 attached cubes on one side balances a train of 3 cubes and a train of 4 cubes on the other side.);

– determine, through investigation using a "balance" model and whole numbers to 10, the number of identical objects that must be added or subtracted to establish equality (*Sample problem:* On a pan balance, 5 cubes are placed on the left side and 8 cubes are placed on the right side. How many cubes should you take off the right side so that both sides balance?).

Grade 1: Data Management and Probability

Overall Expectations

By the end of Grade 1, students will:

- collect and organize categorical primary data and display the data using concrete graphs and pictographs, without regard to the order of labels on the horizontal axis;
- read and describe primary data presented in concrete graphs and pictographs;
- describe the likelihood that everyday events will happen.

Specific Expectations

Collection and Organization of Data

By the end of Grade 1, students will:

– demonstrate an ability to organize objects into categories by sorting and classifying objects using one attribute (e.g., colour, size), and by describing informal sorting experiences (e.g., helping to put away groceries) (*Sample problem:* Sort a collection of attribute blocks by colour. Re-sort the same collection by shape.);

– collect and organize primary data (e.g., data collected by the class) that is categorical (i.e., that can be organized into categories based on qualities such as colour or hobby), and display the data using one-to-one correspondence, prepared templates of concrete graphs and pictographs (with titles and labels), and a variety of recording methods (e.g., arranging objects, placing stickers, drawing pictures, making tally marks) (*Sample problem:* Collect and organize data about the favourite fruit that students in your class like to eat.).

Data Relationships

By the end of Grade 1, students will:

– read primary data presented in concrete graphs and pictographs, and describe the data using comparative language (e.g., more students chose summer than winter as their single favourite season);

– pose and answer questions about collected data (*Sample problem:* What was the most popular fruit chosen by the students in your class?).

Probability

By the end of Grade 1, students will:

– describe the likelihood that everyday events will occur, using mathematical language (i.e., *impossible*, *unlikely*, *less likely*, *more likely*, *certain*) (e.g., "It's unlikely that I will win the contest shown on the cereal box.").

Grade 2

The following are highlights of student learning in Grade 2. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering numbers to 100; representing money amounts to 100¢; decomposing and composing two-digit numbers; investigating fractions of a whole; counting by 1's, 2's, 5's, 10's, and 25's; adding and subtracting two-digit numbers in a variety of ways; relating equal-sized groups to multiplication and relating sharing equally to division

Measurement: measuring length using centimetres and metres; telling time to the nearest quarter-hour; measuring perimeter, area, mass, and capacity using non-standard units; describing and establishing temperature change; choosing personal referents for the centimetre and the metre; comparing the mass and capacity of objects using non-standard units; relating days to weeks and months to years

Geometry and Spatial Sense: distinguishing between attributes that are geometric properties and attributes that are not geometric properties; classifying two-dimensional shapes by geometric properties (number of sides and vertices); classifying three-dimensional figures by geometric properties (number and shape of faces); locating a line of symmetry; composing and decomposing shapes; describing relative locations and paths of motion

Patterning and Algebra: identifying and describing repeating patterns and growing and shrinking patterns; developing the concept of equality using the addition and subtraction of numbers to 18 and the equal sign; using the commutative property and the property of zero in addition to facilitate computation

Data Management and Probability: organizing objects into categories using two attributes; collecting and organizing categorical and discrete data; reading and displaying data using line plots and simple bar graphs; describing probability, in simple games and experiments, as the likelihood that an event will occur

Grade 2: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 2, students will:

PROBLEM SOLVING
- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;

REPRESENTING
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

Grade 2: Number Sense and Numeration

Overall Expectations

By the end of Grade 2, students will:

- read, represent, compare, and order whole numbers to 100, and use concrete materials to represent fractions and money amounts to 100¢;
- demonstrate an understanding of magnitude by counting forward to 200 and backwards from 50, using multiples of various numbers as starting points;
- solve problems involving the addition and subtraction of one- and two-digit whole numbers, using a variety of strategies, and investigate multiplication and division.

Specific Expectations

Quantity Relationships

By the end of Grade 2, students will:

– represent, compare, and order whole numbers to 100, including money amounts to 100¢, using a variety of tools (e.g., ten frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets);

– read and print in words whole numbers to twenty, using meaningful contexts (e.g., storybooks, posters, signs);

– compose and decompose two-digit numbers in a variety of ways, using concrete materials (e.g., place 42 counters on ten frames to show 4 tens and 2 ones; compose 37¢ using one quarter, one dime, and two pennies) (*Sample problem:* Use base ten blocks to show 60 in different ways.);

– determine, using concrete materials, the ten that is nearest to a given two-digit number, and justify the answer (e.g., use counters on ten frames to determine that 47 is closer to 50 than to 40);

– determine, through investigation using concrete materials, the relationship between the number of fractional parts of a whole and the size of the fractional parts (e.g., a paper plate divided into fourths has larger parts than a paper plate divided into eighths) (*Sample problem:* Use paper squares to show which is bigger, one half of a square or one fourth of a square.);

– regroup fractional parts into wholes, using concrete materials (e.g., combine nine fourths to form two wholes and one fourth);

– compare fractions using concrete materials, without using standard fractional notation (e.g., use fraction pieces to show that three fourths are bigger than one half, but smaller than one whole);

– estimate, count, and represent (using the ¢ symbol) the value of a collection of coins with a maximum value of one dollar.

Counting

By the end of Grade 2, students will:

– count forward by 1's, 2's, 5's, 10's, and 25's to 200, using number lines and hundreds charts, starting from multiples of 1, 2, 5, and 10 (e.g., count by 5's from 15; count by 25's from 125);

– count backwards by 1's from 50 and any number less than 50, and count backwards by 10's from 100 and any number less than 100, using number lines and hundreds charts (*Sample problem:* Count backwards from 87 on a hundreds carpet, and describe any patterns you see.);

– locate whole numbers to 100 on a number line and on a partial number line (e.g., locate 37 on a partial number line that goes from 34 to 41).

Operational Sense

By the end of Grade 2, students will:

– solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies (e.g., "To add 6 + 8, I could double 6 and get 12 and then add 2 more to get 14.");

– describe relationships between quantities by using whole-number addition and subtraction (e.g., "If you ate 7 grapes and I ate 12 grapes, I can say that I ate 5 more grapes than you did, or you ate 5 fewer grapes than I did.");

– represent and explain, through investigation using concrete materials and drawings, multiplication as the combining of equal groups (e.g., use counters to show that 3 groups of 2 is equal to 2 + 2 + 2 and to 3 x 2);

– represent and explain, through investigation using concrete materials and drawings, division as the sharing of a quantity equally (e.g., "I can share 12 carrot sticks equally among 4 friends by giving each person 3 carrot sticks.");

– solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (e.g., base ten materials, counters), student-generated algorithms, and standard algorithms;

– add and subtract money amounts to 100¢, using a variety of tools (e.g., concrete materials, drawings) and strategies (e.g., counting on, estimating, representing using symbols).

Grade 2: Measurement

Overall Expectations

By the end of Grade 2, students will:

- estimate, measure, and record length, perimeter, area, mass, capacity, time, and temperature, using non-standard units and standard units;
- compare, describe, and order objects, using attributes measured in non-standard units and standard units.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 2, students will:

– choose benchmarks – in this case, personal referents – for a centimetre and a metre (e.g., "My little finger is about as wide as one centimetre. A really big step is about one metre.") to help them perform measurement tasks;

– estimate and measure length, height, and distance, using standard units (i.e., centimetre, metre) and non-standard units;

– record and represent measurements of length, height, and distance in a variety of ways (e.g., written, pictorial, concrete) (*Sample problem:* Investigate how the steepness of a ramp affects the distance an object travels. Use cash-register tape for recording distances.);

– select and justify the choice of a standard unit (i.e., centimetre or metre) or a non-standard unit to measure length (e.g., "I needed a fast way to check that the two teams would race the same distance, so I used paces.");

– estimate, measure, and record the distance around objects, using non-standard units (*Sample problem:* Measure around several different doll beds using string, to see which bed is the longest around.);

– estimate, measure, and record area, through investigation using a variety of non-standard units (e.g., determine the number of yellow pattern blocks it takes to cover an outlined shape) (*Sample problem:* Cover your desk with index cards in more than one way. See if the number of index cards needed stays the same each time.);

– estimate, measure, and record the capacity and/or mass of an object, using a variety of non-standard units (e.g., "I used the pan balance and found that the stapler has the same mass as my pencil case.");

– tell and write time to the quarter-hour, using demonstration digital and analogue clocks (e.g., "My clock shows the time recess will start [10:00], and my friend's clock shows the time recess will end [10:15].");

– construct tools for measuring time intervals in non-standard units (e.g., a particular bottle of water takes about five seconds to empty);

– describe how changes in temperature affect everyday experiences (e.g., the choice of clothing to wear);

– use a standard thermometer to determine whether temperature is rising or falling (e.g., the temperature of water, air).

Measurement Relationships

By the end of Grade 2, students will:

– describe, through investigation, the relationship between the size of a unit of area and the number of units needed to cover a surface (*Sample problem:* Compare the numbers of hexagon pattern blocks and triangle pattern blocks needed to cover the same book.);

– compare and order a collection of objects by mass and/or capacity, using non-standard units (e.g., "The coffee can holds more sand than the soup can, but the same amount as the small pail.");

– determine, through investigation, the relationship between days and weeks and between months and years.

Grade 2: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 2, students will:

- identify two-dimensional shapes and three-dimensional figures and sort and classify them by their geometric properties;
- compose and decompose two-dimensional shapes and three-dimensional figures;
- describe and represent the relative locations of objects, and represent objects on a map.

Specific Expectations

Geometric Properties

By the end of Grade 2, students will:

– distinguish between the attributes of an object that are geometric properties (e.g., number of sides, number of faces) and the attributes that are not geometric properties (e.g., colour, size, texture), using a variety of tools (e.g., attribute blocks, geometric solids, connecting cubes);

– identify and describe various polygons (i.e., triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons) and sort and classify them by their geometric properties (i.e., number of sides or number of vertices), using concrete materials and pictorial representations (e.g., "I put all the figures with five or more vertices in one group, and all the figures with fewer than five vertices in another group.");

– identify and describe various three-dimensional figures (i.e., cubes, prisms, pyramids) and sort and classify them by their geometric properties (i.e., number and shape of faces), using concrete materials (e.g., "I separated the figures that have square faces from the ones that don't.");

– create models and skeletons of prisms and pyramids, using concrete materials (e.g., cardboard; straws and modelling clay), and describe their geometric properties (i.e., number and shape of faces, number of edges);

– locate the line of symmetry in a two-dimensional shape (e.g., by paper folding; by using a Mira).

Geometric Relationships

By the end of Grade 2, students will:

– compose and describe pictures, designs, and patterns by combining two-dimensional shapes (e.g., "I made a picture of a flower from one hexagon and six equilateral triangles.");

– compose and decompose two-dimensional shapes (*Sample problem:* Use Power Polygons to show if you can compose a rectangle from two triangles of different sizes.);

– cover an outline puzzle with two-dimensional shapes in more than one way;

– build a structure using three-dimensional figures, and describe the two-dimensional shapes and three-dimensional figures in the structure (e.g., "I used a box that looks like a triangular prism to build the roof of my house.").

Location and Movement

By the end of Grade 2, students will:

– describe the relative locations (e.g., beside, two steps to the right of) and the movements of objects on a map (e.g., "The path shows that he walked around the desk, down the aisle, and over to the window.");

– draw simple maps of familiar settings, and describe the relative locations of objects on the maps (*Sample problem:* Draw a map of the classroom, showing the locations of the different pieces of furniture.);
– create and describe symmetrical designs using a variety of tools (e.g., pattern blocks, tangrams, paper and pencil).

Grade 2: Patterning and Algebra

Overall Expectations

By the end of Grade 2, students will:

- identify, describe, extend, and create repeating patterns, growing patterns, and shrinking patterns;
- demonstrate an understanding of the concept of equality between pairs of expressions, using concrete materials, symbols, and addition and subtraction to 18.

Specific Expectations

Patterns and Relationships

By the end of Grade 2, students will:

– identify and describe, through investigation, growing patterns and shrinking patterns generated by the repeated addition or subtraction of 1's, 2's, 5's, 10's, and 25's on a number line and on a hundreds chart (e.g., the numbers 90, 80, 70, 60, 50, 40, 30, 20, 10 are in a straight line on a hundreds chart);

– identify, describe, and create, through investigation, growing patterns and shrinking patterns involving addition and subtraction, with and without the use of calculators (e.g., 3 + 1 = 4, 3 + 2 = 5, 3 + 3 = 6, …);

– identify repeating, growing, and shrinking patterns found in real-life contexts (e.g., a geometric pattern on wallpaper, a rhythm pattern in music, a number pattern when counting dimes);

– represent a given growing or shrinking pattern in a variety of ways (e.g., using pictures, actions, colours, sounds, numbers, letters, number lines, bar graphs) (*Sample problem:* Show the letter pattern A, AA, AAA, AAAA, … by clapping or hopping.);

– create growing or shrinking patterns (*Sample problem:* Create a shrinking pattern using cut-outs of pennies and/or nickels, starting with 20 cents.);

– create a repeating pattern by combining two attributes (e.g., colour and shape; colour and size) (*Sample problem:* Use attribute blocks to make a train that shows a repeating pattern involving two attributes.);

– demonstrate, through investigation, an understanding that a pattern results from repeating an operation (e.g., addition, subtraction) or making a repeated change to an attribute (e.g., colour, orientation).

Expressions and Equality

By the end of Grade 2, students will:

– demonstrate an understanding of the concept of equality by partitioning whole numbers to 18 in a variety of ways, using concrete materials (e.g., starting with 9 tiles and adding 6 more tiles gives the same result as starting with 10 tiles and adding 5 more tiles);

– represent, through investigation with concrete materials and pictures, two number expressions that are equal, using the equal sign (e.g., "I can break a train of 10 cubes into 4 cubes and 6 cubes. I can also break 10 cubes into 7 cubes and 3 cubes. This means 4 + 6 = 7 + 3.");

– determine the missing number in equations involving addition and subtraction to 18, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (*Sample problem:* Use counters to determine the missing number in the equation 6 + 7 = ☐ + 5.);

– identify, through investigation, and use the commutative property of addition (e.g., create a train of 10 cubes by joining 4 red cubes to 6 blue cubes, or by joining 6 blue cubes to 4 red cubes) to facilitate computation with whole numbers (e.g., "I know that 9 + 8 + 1 = 9 + 1 + 8. Adding becomes easier because that gives 10 + 8 = 18.");

– identify, through investigation, the properties of zero in addition and subtraction (i.e., when you add zero to a number, the number does not change; when you subtract zero from a number, the number does not change).

Grade 2: Data Management and Probability

Overall Expectations

By the end of Grade 2, students will:

- collect and organize categorical or discrete primary data and display the data, using tally charts, concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers, with labels ordered appropriately along horizontal axes, as needed;
- read and describe primary data presented in tally charts, concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers;
- describe probability in everyday situations and simple games.

Specific Expectations

Collection and Organization of Data

By the end of Grade 2, students will:

– demonstrate an ability to organize objects into categories, by sorting and classifying objects using two attributes simultaneously (e.g., sort attribute blocks by colour and shape at the same time);

– gather data to answer a question, using a simple survey with a limited number of responses (e.g., What is your favourite season?; How many letters are in your first name?);

– collect and organize primary data (e.g., data collected by the class) that is categorical or discrete (i.e., that can be counted, such as the number of students absent), and display the data using one-to-one correspondence in concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers (e.g., tally charts, diagrams), with appropriate titles and labels and with labels ordered appropriately along horizontal axes, as needed (*Sample problem:* Record the number of times that specific words are used in a simple rhyme or poem.).

Data Relationships

By the end of Grade 2, students will:

– read primary data presented in concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers (e.g., tally charts, diagrams), and describe the data using mathematical language (e.g., "Our bar graph shows that 4 more students walk to school than take the bus.");

– pose and answer questions about class-generated data in concrete graphs, pictographs, line plots, simple bar graphs, and tally charts (e.g., Which is the least favourite season?);

– distinguish between numbers that represent data values (e.g., "I have 4 people in my family.") and numbers that represent the frequency of an event (e.g., "There are 10 children in my class who have 4 people in their family.");

– demonstrate an understanding of data displayed in a graph (e.g., by telling a story, by drawing a picture), by comparing different parts of the data and by making statements about the data as a whole (e.g., "I looked at the graph that shows how many students were absent each month. More students were away in January than in September.").

Probability

By the end of Grade 2, students will:

– describe probability as a measure of the likelihood that an event will occur, using mathematical language (i.e., *impossible*, *unlikely*, *less likely*, *equally likely*, *more likely*, *certain*) (e.g., "If I take a new shoe out of a box without looking, it's equally likely that I will pick the left shoe or the right shoe.");

– describe the probability that an event will occur (e.g., getting heads when tossing a coin, landing on red when spinning a spinner), through investigation with simple games and probability experiments and using mathematical language (e.g., "I tossed 2 coins at the same time, to see how often I would get 2 heads. I found that getting a head and a tail was more likely than getting 2 heads.") (*Sample problem:* Describe the probability of spinning red when you spin a spinner that has one half shaded yellow, one fourth shaded blue, and one fourth shaded red. Experiment with the spinner to see if the results are what you expected.).

Grade 3

The following are highlights of student learning in Grade 3. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering numbers to 1000; representing money amounts to $10; decomposing and composing three-digit numbers; investigating fractions of a set; counting by 1's, 2's, 5's, 10's, 25's, and 100's; adding and subtracting three-digit numbers in a variety of ways; relating one-digit multiplication, and division by one-digit divisors, to real-life situations

Measurement: measuring distance using kilometres; telling time to the nearest 5 minutes; identifying temperature benchmarks; measuring perimeter using standard units; measuring mass in kilograms and capacity in litres; measuring area using grid paper; comparing the length, mass, and capacity of objects using standard units; relating minutes to hours, hours to days, days to weeks, and weeks to years

Geometry and Spatial Sense: using a reference tool to identify right angles and to compare angles with a right angle; classifying two-dimensional shapes by geometric properties (number of sides and angles); classifying three-dimensional figures by geometric properties (number of faces, edges, and vertices); relating different types of quadrilaterals; naming prisms and pyramids; identifying congruent shapes; describing movement on a grid map; recognizing transformations

Patterning and Algebra: creating and extending growing and shrinking patterns; representing geometric patterns with a number sequence, a number line, and a bar graph; determining the missing numbers in equations involving addition and subtraction of one- and two-digit numbers; investigating the properties of zero and one in multiplication

Data Management and Probability: organizing objects into categories using two or more attributes; collecting and organizing categorical and discrete data; reading and displaying data using vertical and horizontal bar graphs; understanding mode; predicting the frequency of an outcome; relating fair games to equally likely events

Grade 3: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 3, students will:

PROBLEM SOLVING
- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;

REPRESENTING
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

Grade 3: Number Sense and Numeration

Overall Expectations

By the end of Grade 3, students will:

- read, represent, compare, and order whole numbers to 1000, and use concrete materials to represent fractions and money amounts to $10;
- demonstrate an understanding of magnitude by counting forward and backwards by various numbers and from various starting points;
- solve problems involving the addition and subtraction of single- and multi-digit whole numbers, using a variety of strategies, and demonstrate an understanding of multiplication and division.

Specific Expectations

Quantity Relationships

By the end of Grade 3, students will:

– represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts);

– read and print in words whole numbers to one hundred, using meaningful contexts (e.g., books, speed limit signs);

– identify and represent the value of a digit in a number according to its position in the number (e.g., use base ten materials to show that the 3 in 324 represents 3 hundreds);

– compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials (e.g., use base ten materials to decompose 327 into 3 hundreds, 2 tens, and 7 ones, or into 2 hundreds, 12 tens, and 7 ones);

– round two-digit numbers to the nearest ten, in problems arising from real-life situations;

– represent and explain, using concrete materials, the relationship among the numbers 1, 10, 100, and 1000, (e.g., use base ten materials to represent the relationship between a decade and a century, or a century and a millennium);

– divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation;

– represent and describe the relationships between coins and bills up to $10 (e.g., "There are eight quarters in a toonie and ten dimes in a loonie.");

– estimate, count, and represent (using the $ symbol) the value of a collection of coins and bills with a maximum value of $10;

– solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1000 (*Sample problem:* Do you know anyone who has lived for close to 1000 days? Explain your reasoning.).

Counting

By the end of Grade 3, students will:

– count forward by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points, and by 25's to 1000 starting from multiples of 25, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10's using dimes);

– count backwards by 2’s, 5’s, and 10’s from 100 using multiples of 2, 5, and 10 as starting points, and count backwards by 100’s from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies.

Operational Sense

By the end of Grade 3, students will:

– solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies (e.g., to add 37 + 26, add the tens, add the ones, then combine the tens and ones, like this: 30 + 20 = 50, 7 + 6 = 13, 50 + 13 = 63);
– add and subtract three-digit numbers, using concrete materials, student-generated algorithms, and standard algorithms;
– use estimation when solving problems involving addition and subtraction, to help judge the reasonableness of a solution;
– add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10 (*Sample problem:* You spent 5 dollars and 75 cents on one item and 10 cents on another item. How much did you spend in total?);
– relate multiplication of one-digit numbers and division by one-digit divisors to real-life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences) (*Sample problem:* Give a real-life example of when you might need to know that 3 groups of 2 is 3 x 2.);
– multiply to 7 x 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting).

Grade 3: Measurement

Overall Expectations

By the end of Grade 3, students will:

- estimate, measure, and record length, perimeter, area, mass, capacity, time, and temperature, using standard units;
- compare, describe, and order objects, using attributes measured in standard units.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 3, students will:

– estimate, measure, and record length, height, and distance, using standard units (i.e., centimetre, metre, kilometre) (*Sample problem:* While walking with your class, stop when you think you have travelled one kilometre.);

– draw items using a ruler, given specific lengths in centimetres (*Sample problem:* Draw a pencil that is 5 cm long);

– read time using analogue clocks, to the nearest five minutes, and using digital clocks (e.g., 1:23 means twenty-three minutes after one o'clock), and represent time in 12-hour notation;

– estimate, read (i.e., using a thermometer), and record positive temperatures to the nearest degree Celsius (i.e., using a number line; using appropriate notation) (*Sample problem:* Record the temperature outside each day using a thermometer, and compare your measurements with those reported in the daily news.);

– identify benchmarks for freezing, cold, cool, warm, hot, and boiling temperatures as they relate to water and for cold, cool, warm, and hot temperatures as they relate to air (e.g., water freezes at 0°C; the air temperature on a warm day is about 20°C, but water at 20°C feels cool);

– estimate, measure, and record the perimeter of two-dimensional shapes, through investigation using standard units (*Sample problem:* Estimate, measure, and record the perimeter of your notebook.);

– estimate, measure (i.e., using centimetre grid paper, arrays), and record area (e.g., if a row of 10 connecting cubes is approximately the width of a book, skip counting down the cover of the book with the row of cubes [i.e., counting 10, 20, 30, ...] is one way to determine the area of the book cover);

– choose benchmarks for a kilogram and a litre to help them perform measurement tasks;

– estimate, measure, and record the mass of objects (e.g., can of apple juice, bag of oranges, bag of sand), using the standard unit of the kilogram or parts of a kilogram (e.g., half, quarter);

– estimate, measure, and record the capacity of containers (e.g., juice can, milk bag), using the standard unit of the litre or parts of a litre (e.g., half, quarter).

Measurement Relationships

By the end of Grade 3, students will:

– compare standard units of length (i.e., centimetre, metre, kilometre) (e.g., centimetres are smaller than metres), and select and justify the most appropriate standard unit to measure length;

– compare and order objects on the basis of linear measurements in centimetres and/or metres (e.g., compare a 3 cm object with a 5 cm object; compare a 50 cm object with a 1 m object) in problem-solving contexts;
– compare and order various shapes by area, using congruent shapes (e.g., from a set of pattern blocks or Power Polygons) and grid paper for measuring (*Sample problem:* Does the order of the shapes change when you change the size of the pattern blocks you measure with?);
– describe, through investigation using grid paper, the relationship between the size of a unit of area and the number of units needed to cover a surface (*Sample problem:* What is the difference between the numbers of squares needed to cover the front of a book, using centimetre grid paper and using two-centimetre grid paper?);
– compare and order a collection of objects, using standard units of mass (i.e., kilogram) and/or capacity (i.e., litre);
– solve problems involving the relationships between minutes and hours, hours and days, days and weeks, and weeks and years, using a variety of tools (e.g., clocks, calendars, calculators).

Grade 3: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 3, students will:

- compare two-dimensional shapes and three-dimensional figures and sort them by their geometric properties;
- describe relationships between two-dimensional shapes, and between two-dimensional shapes and three-dimensional figures;
- identify and describe the locations and movements of shapes and objects.

Specific Expectations

Geometric Properties

By the end of Grade 3, students will:

– use a reference tool (e.g., paper corner, pattern block, carpenter's square) to identify right angles and to describe angles as greater than, equal to, or less than a right angle (*Sample problem:* Which pattern blocks have angles bigger than a right angle?);

– identify and compare various polygons (i.e., triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons) and sort them by their geometric properties (i.e., number of sides; side lengths; number of interior angles; number of right angles);

– compare various angles, using concrete materials and pictorial representations, and describe angles as *bigger than*, *smaller than*, or *about the same as* other angles (e.g., "Two of the angles on the red pattern block are bigger than all the angles on the green pattern block.");

– compare and sort prisms and pyramids by geometric properties (i.e., number and shape of faces, number of edges, number of vertices), using concrete materials;

– construct rectangular prisms (e.g., using given paper nets; using Polydrons), and describe geometric properties (i.e., number and shape of faces, number of edges, number of vertices) of the prisms.

Geometric Relationships

By the end of Grade 3, students will:

– solve problems requiring the greatest or least number of two-dimensional shapes (e.g., pattern blocks) needed to compose a larger shape in a variety of ways (e.g., to cover an outline puzzle) (*Sample problem:* Compose a hexagon using different numbers of smaller shapes.);

– explain the relationships between different types of quadrilaterals (e.g., a square is a rectangle because a square has four sides and four right angles; a rhombus is a parallelogram because opposite sides of a rhombus are parallel);

– identify and describe the two-dimensional shapes that can be found in a three-dimensional figure (*Sample problem:* Build a structure from blocks, toothpicks, or other concrete materials, and describe it using geometric terms, so that your partner will be able to build your structure without seeing it.);

– describe and name prisms and pyramids by the shape of their base (e.g., rectangular prism, square-based pyramid);

– identify congruent two-dimensional shapes by manipulating and matching concrete materials (e.g., by translating, reflecting, or rotating pattern blocks).

Location and Movement

By the end of Grade 3, students will:

– describe movement from one location to another using a grid map (e.g., to get from the swings to the sandbox, move three squares to the right and two squares down);

– identify flips, slides, and turns, through investigation using concrete materials and physical motion, and name flips, slides, and turns as reflections, translations, and rotations (e.g., a slide to the right is a translation; a turn is a rotation);

– complete and describe designs and pictures of images that have a vertical, horizontal, or diagonal line of symmetry (*Sample problem:* Draw the missing portion of the given butterfly on grid paper.).

Grade 3: Patterning and Algebra

Overall Expectations

By the end of Grade 3, students will:

- describe, extend, and create a variety of numeric patterns and geometric patterns;
- demonstrate an understanding of equality between pairs of expressions, using addition and subtraction of one- and two-digit numbers.

Specific Expectations

Patterns and Relationships

By the end of Grade 3, students will:

– identify, extend, and create a repeating pattern involving two attributes (e.g., size, colour, orientation, number), using a variety of tools (e.g., pattern blocks, attribute blocks, drawings) (*Sample problem:* Create a repeating pattern using three colours and two shapes.);

– identify and describe, through investigation, number patterns involving addition, subtraction, and multiplication, represented on a number line, on a calendar, and on a hundreds chart (e.g., the multiples of 9 appear diagonally in a hundreds chart);

– extend repeating, growing, and shrinking number patterns (*Sample problem:* Write the next three terms in the pattern 4, 8, 12, 16, ….);

– create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words (*Sample problem:* Make a number pattern that starts at 0 and grows by adding 7 each time.);

– represent simple geometric patterns using a number sequence, a number line, or a bar graph (e.g., the given growing pattern of toothpick squares can be represented numerically by the sequence 4, 7, 10, …, which represents the number of toothpicks used to make each figure);

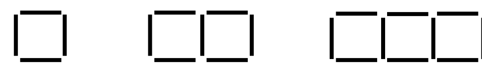


Figure 1 Figure 2 Figure 3

– demonstrate, through investigation, an understanding that a pattern results from repeating an action (e.g., clapping, taking a step forward every second), repeating an operation (e.g., addition, subtraction), using a transformation (e.g., slide, flip, turn), or making some other repeated change to an attribute (e.g., colour, orientation).

Expressions and Equality

By the end of Grade 3, students will:

– determine, through investigation, the inverse relationship between addition and subtraction (e.g., since 4 + 5 = 9, then 9 – 5 = 4; since 16 – 9 = 7, then 7 + 9 = 16);

– determine, the missing number in equations involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (*Sample problem:* What is the missing number in the equation 25 – 4 = 15 + ☐?);

– identify, through investigation, the properties of zero and one in multiplication (i.e., any number multiplied by zero equals zero; any number multiplied by 1 equals the original number) (***Sample problem:*** Use tiles to create arrays that represent 3 x 3, 3 x 2, 3 x 1, and 3 x 0. Explain what you think will happen when you multiply any number by 1, and when you multiply any number by 0.);

– identify, through investigation, and use the associative property of addition to facilitate computation with whole numbers (e.g., "I know that 17 + 16 equals 17 + 3 + 13. This is easier to add in my head because I get 20 + 13 = 33.").

Grade 3: Data Management and Probability

Overall Expectations

By the end of Grade 3, students will:

- collect and organize categorical or discrete primary data and display the data using charts and graphs, including vertical and horizontal bar graphs, with labels ordered appropriately along horizontal axes, as needed;
- read, describe, and interpret primary data presented in charts and graphs, including vertical and horizontal bar graphs;
- predict and investigate the frequency of a specific outcome in a simple probability experiment.

Specific Expectations

Collection and Organization of Data

By the end of Grade 3, students will:

– demonstrate an ability to organize objects into categories, by sorting and classifying objects using two or more attributes simultaneously (*Sample problem:* Sort a collection of buttons by size, colour, and number of holes.);

– collect data by conducting a simple survey about themselves, their environment, issues in their school or community, or content from another subject;

– collect and organize categorical or discrete primary data and display the data in charts, tables, and graphs (including vertical and horizontal bar graphs), with appropriate titles and labels and with labels ordered appropriately along horizontal axes, as needed, using many-to-one correspondence (e.g., in a pictograph, one car sticker represents 3 cars; on a bar graph, one square represents 2 students) (*Sample problem:* Graph data related to the eye colour of students in the class, using a vertical bar graph. Why does the scale on the vertical axis include values that are not in the set of data?).

Data Relationships

By the end of Grade 3, students will:

– read primary data presented in charts, tables, and graphs (including vertical and horizontal bar graphs), then describe the data using comparative language, and describe the shape of the data (e.g., "Most of the data are at the high end."; "All of the data values are different.");

– interpret and draw conclusions from data presented in charts, tables, and graphs;

– demonstrate an understanding of mode (e.g., "The mode is the value that shows up most often on a graph."), and identify the mode in a set of data.

Probability

By the end of Grade 3, students will:

– predict the frequency of an outcome in a simple probability experiment or game (e.g., "I predict that an even number will come up 5 times and an odd number will come up 5 times when I roll a number cube 10 times."), then perform the experiment, and compare the results with the predictions, using mathematical language;

– demonstrate, through investigation, an understanding of fairness in a game and relate this to the occurrence of equally likely outcomes.

Grade 4

The following are highlights of student learning in Grade 4. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering numbers to 10 000; representing money amounts to $100; developing the concept of place value to tenths; representing and comparing fractions using fractional notation; adding and subtracting three-digit numbers in a variety of ways; multiplying and dividing two-digit whole numbers by one-digit whole numbers; relating halves, fifths, and tenths to decimals

Measurement: measuring length using millimetres; measuring time intervals to the nearest minute; determining elapsed time; measuring mass in grams and capacity in millilitres; measuring volume using concrete materials; determining area and perimeter relationships for rectangles; comparing the mass and capacity of objects using standard units; relating years to decades and decades to centuries

Geometry and Spatial Sense: identifying geometric properties of parallelograms; classifying two-dimensional shapes by geometric properties (number of sides, angles, and symmetry); identifying a straight angle, a right angle, and half a right angle; classifying prisms and pyramids by geometric properties; constructing three-dimensional figures in a variety of ways; describing location using a grid system; performing and describing reflections

Patterning and Algebra: relating the term and the term number in a numeric sequence; generating patterns that involve addition, subtraction, multiplication, and reflections; determining the missing numbers in equations involving multiplication of one- and two-digit numbers; using the commutative and distributive properties to facilitate computation

Data Management and Probability: collecting and organizing discrete data; reading and displaying data using stem-and-leaf plots and double bar graphs; understanding median; comparing two related sets of data; predicting the frequency of an outcome; investigating how the number of repetitions of a probability experiment affects the conclusion drawn

Grade 4: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 4, students will:

PROBLEM SOLVING
- develop, select, and apply problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counter-examples) to make and investigate conjectures and construct and defend arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., by using physical models, pictures, numbers, variables, diagrams, graphs, onscreen dynamic representations), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, and observing basic mathematical conventions.

Grade 4: Number Sense and Numeration

Overall Expectations

By the end of Grade 4, students will:

- read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;
- demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;
- solve problems involving the addition, subtraction, multiplication, and division of single- and multi-digit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies;
- demonstrate an understanding of proportional reasoning by investigating whole-number unit rates.

Specific Expectations

Quantity Relationships

By the end of Grade 4, students will:

– represent, compare, and order whole numbers to 10 000, using a variety of tools (e.g., drawings of base ten materials, number lines with increments of 100 or other appropriate amounts);

– demonstrate an understanding of place value in whole numbers and decimal numbers from 0.1 to 10 000, using a variety of tools and strategies (e.g., use base ten materials to represent 9307 as 9000 + 300 + 0 + 7) (*Sample problem:* Use the digits 1, 9, 5, 4 to create the greatest number and the least number possible, and explain your thinking.);

– read and print in words whole numbers to one thousand, using meaningful contexts (e.g., books, highway distance signs);

– round four-digit whole numbers to the nearest ten, hundred, and thousand, in problems arising from real-life situations;

– represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation (*Sample problem:* Draw a partial number line that extends from 4.2 to 6.7, and mark the location of 5.6.);

– represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;

– compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts (e.g., $\frac{4}{5}$ is greater than $\frac{3}{5}$ because there are more parts in $\frac{4}{5}$; $\frac{1}{4}$ is greater than $\frac{1}{5}$ because the size of the part is larger in $\frac{1}{4}$);

– compare fractions to the benchmarks of 0, $\frac{1}{2}$, and 1 (e.g., $\frac{1}{8}$ is closer to 0 than to $\frac{1}{2}$; $\frac{3}{5}$ is more than $\frac{1}{2}$);

– demonstrate and explain the relationship between equivalent fractions, using concrete materials (e.g., fraction circles, fraction strips, pattern blocks) and drawings (e.g., "I can say that $\frac{3}{6}$ of my cubes are white, or half of the cubes are white. This means that $\frac{3}{6}$ and $\frac{1}{2}$ are equal.");
– read and represent money amounts to $100 (e.g., five dollars, two quarters, one nickel, and four cents is $5.59);
– solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 10 000 (*Sample problem:* How high would a stack of 10 000 pennies be? Justify your answer.).

Counting

By the end of Grade 4, students will:

– count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines (e.g., use fraction circles to count fourths: "One fourth, two fourths, three fourths, four fourths, five fourths, six fourths, …");
– count forward by tenths from any decimal number expressed to one decimal place, using concrete materials and number lines (e.g., use base ten materials to represent 3.7 and count forward: 3.8, 3.9, 4.0, 4.1, …; "Three and seven tenths, three and eight tenths, three and nine tenths, four, four and one tenth, …") (*Sample problem:* What connections can you make between counting by tenths and measuring lengths in millimetres and in centimetres?).

Operational Sense

By the end of Grade 4, students will:

– add and subtract two-digit numbers, using a variety of mental strategies (e.g., one way to calculate 73 – 39 is to subtract 40 from 73 to get 33, and then add 1 back to get 34);
– solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms (e.g., "I added 4217 + 1914 using 5000 + 1100 + 20 + 11.");
– add and subtract decimal numbers to tenths, using concrete materials (e.g., paper strips divided into tenths, base ten materials) and student-generated algorithms (e.g., "When I added 6.5 and 5.6, I took five tenths in fraction circles and added six tenths in fraction circles to give me one whole and one tenth. Then I added 6 + 5 + 1.1, which equals 12.1.");
– add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools (e.g., currency manipulatives, drawings);
– multiply to 9 x 9 and divide to 81 ÷ 9, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting);
– solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies (e.g., 6 x 8 can be thought of as 5 x 8 + 1 x 8);
– multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);
– multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms;
– divide two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms;

– use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution (*Sample problem:* A school is ordering pencils that come in boxes of 100. If there are 9 classes and each class needs about 110 pencils, estimate how many boxes the school should buy.).

Proportional Relationships

By the end of Grade 4, students will:

– describe relationships that involve simple whole-number multiplication (e.g., "If you have 2 marbles and I have 6 marbles, I can say that I have three times the number of marbles you have.");

– determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., decompose $\frac{2}{5}$ into $\frac{4}{10}$ by dividing each fifth into two equal parts to show that $\frac{2}{5}$ can be represented as 0.4);

– demonstrate an understanding of simple multiplicative relationships involving unit rates, through investigation using concrete materials and drawings (e.g., scale drawings in which 1 cm represents 2 m) (*Sample problem:* If 1 book costs $4, how do you determine the cost of 2 books? … 3 books? … 4 books?).

Grade 4: Measurement

Overall Expectations

By the end of Grade 4, students will:

- estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies;
- determine the relationships among units and measurable attributes, including the area and perimeter of rectangles.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 4, students will:

– estimate, measure, and record length, height, and distance, using standard units (i.e., millimetre, centimetre, metre, kilometre) (e.g., a pencil that is 75 mm long);

– draw items using a ruler, given specific lengths in millimetres or centimetres (*Sample problem:* Use estimation to draw a line that is 115 mm long. Beside it, use a ruler to draw a line that is 115 mm long. Compare the lengths of the lines.);

– estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest minute;

– estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in five-minute intervals, hours, days, weeks, months, or years (*Sample problem:* If you wake up at 7:30 a.m., and it takes you 10 minutes to eat your breakfast, 5 minutes to brush your teeth, 25 minutes to wash and get dressed, 5 minutes to get your backpack ready, and 20 minutes to get to school, will you be at school by 9:00 a.m.?);

– estimate, measure using a variety of tools (e.g., centimetre grid paper, geoboard) and strategies, and record the perimeter and area of polygons;

– estimate, measure, and record the mass of objects (e.g., apple, baseball, book), using the standard units of the kilogram and the gram;

– estimate, measure, and record the capacity of containers (e.g., a drinking glass, a juice box), using the standard units of the litre and the millilitre;

– estimate, measure using concrete materials, and record volume, and relate volume to the space taken up by an object (e.g., use centimetre cubes to demonstrate how much space a rectangular prism takes up) (*Sample problem:* Build a rectangular prism using connecting cubes. Describe the volume of the prism using the number of connecting cubes.).

Measurement Relationships

By the end of Grade 4, students will:

– describe, through investigation, the relationship between various units of length (i.e., millimetre, centimetre, decimetre, metre, kilometre);

– select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure the side lengths and perimeters of various polygons;

– determine, through investigation, the relationship between the side lengths of a rectangle and its perimeter and area (*Sample problem:* Create a variety of rectangles on a geoboard. Record the length, width, area, and perimeter of each rectangle on a chart. Identify relationships.);
– pose and solve meaningful problems that require the ability to distinguish perimeter and area (e.g., "I need to know about area when I cover a bulletin board with construction paper. I need to know about perimeter when I make the border.");
– compare and order a collection of objects, using standard units of mass (i.e., gram, kilogram) and/or capacity (i.e., millilitre, litre);
– determine, through investigation, the relationship between grams and kilograms (*Sample problem:* Use centimetre cubes with a mass of one gram, or other objects of known mass, to balance a one-kilogram mass.);
– determine, through investigation, the relationship between millilitres and litres (*Sample problem:* Use small containers of different known capacities to fill a one-litre container.);
– select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram) and the most appropriate standard unit to measure the capacity of a container (i.e., millilitre, litre);
– solve problems involving the relationship between years and decades, and between decades and centuries (*Sample problem:* How many decades old is Canada?);
– compare, using a variety of tools (e.g., geoboard, patterns blocks, dot paper), two-dimensional shapes that have the same perimeter or the same area (*Sample problem:* Draw, using grid paper, as many different rectangles with a perimeter of 10 units as you can make on a geoboard.).

Grade 4: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 4, students will:

- identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks;
- construct three-dimensional figures, using two-dimensional shapes;
- identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes.

Specific Expectations

Geometric Properties

By the end of Grade 4, students will:

– draw the lines of symmetry of two-dimensional shapes, through investigation using a variety of tools (e.g., Mira, grid paper) and strategies (e.g., paper folding) (*Sample problem:* Use paper folding to compare the symmetry of a rectangle with the symmetry of a square.);

– identify and compare different types of quadrilaterals (i.e., rectangle, square, trapezoid, parallelogram, rhombus) and sort and classify them by their geometric properties (e.g., sides of equal length; parallel sides; symmetry; number of right angles);

– identify benchmark angles (i.e., straight angle, right angle, half a right angle), using a reference tool (e.g., paper and fasteners, pattern blocks, straws), and compare other angles to these benchmarks (e.g., "The angle the door makes with the wall is smaller than a right angle but greater than half a right angle.") (*Sample problem:* Use paper folding to create benchmarks for a straight angle, a right angle, and half a right angle, and use these benchmarks to describe angles found in pattern blocks.);

– relate the names of the benchmark angles to their measures in degrees (e.g., a right angle is 90°);

– identify and describe prisms and pyramids, and classify them by their geometric properties (i.e., shape of faces, number of edges, number of vertices), using concrete materials.

Geometric Relationships

By the end of Grade 4, students will:

– construct a three-dimensional figure from a picture or model of the figure, using connecting cubes (e.g., use connecting cubes to construct a rectangular prism);

– construct skeletons of three-dimensional figures, using a variety of tools (e.g., straws and modelling clay, toothpicks and marshmallows, Polydrons), and sketch the skeletons;

– draw and describe nets of rectangular and triangular prisms (*Sample problem:* Create as many different nets for a cube as you can, and share your results with a partner.);

– construct prisms and pyramids from given nets;

– construct three-dimensional figures (e.g., cube, tetrahedron), using only congruent shapes.

Location and Movement

By the end of Grade 4, students will:

– identify and describe the general location of an object using a grid system (e.g., "The library is located at A3 on the map.");

– identify, perform, and describe reflections using a variety of tools (e.g., Mira, dot paper, technology);
– create and analyse symmetrical designs by reflecting a shape, or shapes, using a variety of tools (e.g., pattern blocks, Mira, geoboard, drawings), and identify the congruent shapes in the designs.

Grade 4: Patterning and Algebra

Overall Expectations

By the end of Grade 4, students will:

- describe, extend, and create a variety of numeric and geometric patterns, make predictions related to the patterns, and investigate repeating patterns involving reflections;
- demonstrate an understanding of equality between pairs of expressions, using addition, subtraction, and multiplication.

Specific Expectations

Patterns and Relationships

By the end of Grade 4, students will:

– extend, describe, and create repeating, growing, and shrinking number patterns (e.g., "I created the pattern 1, 3, 4, 6, 7, 9, …. I started at 1, then added 2, then added 1, then added 2, then added 1, and I kept repeating this.");

– connect each term in a growing or shrinking pattern with its term number (e.g., in the sequence 1, 4, 7, 10, …, the first term is 1, the second term is 4, the third term is 7, and so on), and record the patterns in a table of values that shows the term number and the term;

– create a number pattern involving addition, subtraction, or multiplication, given a pattern rule expressed in words (e.g., the pattern rule "start at 1 and multiply each term by 2 to get the next term" generates the sequence 1, 2, 4, 8, 16, 32, 64, …);

– make predictions related to repeating geometric and numeric patterns (*Sample problem:* Create a pattern block train by alternating one green triangle with one red trapezoid. Predict which block will be in the 30th place.);

– extend and create repeating patterns that result from reflections, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, dot paper).

Expressions and Equality

By the end of Grade 4, students will:

– determine, through investigation, the inverse relationship between multiplication and division (e.g., since 4 x 5 = 20, then 20 ÷ 5 = 4; since 35 ÷ 5 = 7, then 7 x 5 = 35);

– determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (*Sample problem:* What is the missing number in the equation ☐ x 4 = 24?);

– identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the commutative property of multiplication to facilitate computation with whole numbers (e.g., "I know that 15 x 7 x 2 equals 15 x 2 x 7. This is easier to multiply in my head because I get 30 x 7 = 210.");

– identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the distributive property of multiplication over addition to facilitate computation with whole numbers (e.g., "I know that 9 x 52 equals 9 x 50 + 9 x 2. This is easier to calculate in my head because I get 450 + 18 = 468.").

Grade 4: Data Management and Probability

Overall Expectations

By the end of Grade 4, students will:

- collect and organize discrete primary data and display the data using charts and graphs, including stem-and-leaf plots and double bar graphs;
- read, describe, and interpret primary data and secondary data presented in charts and graphs, including stem-and-leaf plots and double bar graphs;
- predict the results of a simple probability experiment, then conduct the experiment and compare the prediction to the results.

Specific Expectations

Collection and Organization of Data

By the end of Grade 4, students will:

– collect data by conducting a survey (e.g., “Choose your favourite meal from the following list: breakfast, lunch, dinner, other.”) or an experiment to do with themselves, their environment, issues in their school or the community, or content from another subject, and record observations or measurements;

– collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, simple spreadsheets, dynamic statistical software).

Data Relationships

By the end of Grade 4, students will:

– read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., temperature data in the newspaper, data from the Internet about endangered species), presented in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs);

– demonstrate, through investigation, an understanding of median (e.g.,“The median is the value in the middle of the data. If there are two middle values, you have to calculate the middle of those two values.”), and determine the median of a set of data (e.g.,“I used a stem-and-leaf plot to help me find the median.”);

– describe the shape of a set of data across its range of values, using charts, tables, and graphs (e.g. “The data values are spread out evenly.”; “The set of data bunches up around the median.”);

– compare similarities and differences between two related sets of data, using a variety of strategies (e.g., by representing the data using tally charts, stem-and-leaf plots, or double bar graphs; by determining the mode or the median; by describing the shape of a data set across its range of values).

Probability

By the end of Grade 4, students will:

– predict the frequency of an outcome in a simple probability experiment, explaining their reasoning; conduct the experiment; and compare the result with the prediction (*Sample problem:* If you toss a pair of number cubes 20 times and calculate the sum for each toss, how many times would you expect to get 12? 7? 1? Explain your thinking. Then conduct the experiment and compare the results with your predictions.);

– determine, through investigation, how the number of repetitions of a probability experiment can affect the conclusions drawn (*Sample problem:* Each student in the class tosses a coin 10 times and records how many times tails comes up. Combine the individual student results to determine a class result, and then compare the individual student results and the class result.).

Grade 5

The following are highlights of student learning in Grade 5. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering numbers to 100 000; representing money amounts to $1000; developing the concept of place value to hundredths; comparing and ordering fractional amounts with like denominators; adding and subtracting decimal amounts to hundredths; multiplying two-digit whole numbers by two-digit whole numbers; dividing three-digit whole numbers by one-digit whole numbers; relating simple fractions to decimals

Measurement: measuring time intervals to the nearest second; determining elapsed time; measuring temperature; converting from metres to centimetres and from kilometres to metres; relating the 12-hour clock to the 24-hour clock; developing and applying area and perimeter relationships for a rectangle; relating capacity and volume; developing and applying the volume relationship for a right rectangular prism

Geometry and Spatial Sense: distinguishing among polygons and among prisms; identifying acute, right, obtuse, and straight angles; measuring angles to 90° with a protractor; constructing triangles; constructing nets of prisms and pyramids; locating objects using the cardinal directions; performing and describing translations

Patterning and Algebra: representing a pattern using a table of values; predicting terms in a pattern; determining the missing numbers in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers; investigating variables as unknown quantities; demonstrating equality using multiplication or division in equations with unknown quantities on both sides

Data Management and Probability: collecting and organizing discrete and continuous data; displaying data using broken-line graphs; sampling data from a population; understanding mean; comparing two related sets of data; representing probability using fractions

Grade 5: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 5, students will:

PROBLEM SOLVING
- develop, select, and apply problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counter-examples) to make and investigate conjectures and construct and defend arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., by using physical models, pictures, numbers, variables, diagrams, graphs, onscreen dynamic representations), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, and observing basic mathematical conventions.

Grade 5: Number Sense and Numeration

Overall Expectations

By the end of Grade 5, students will:

- read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;
- demonstrate an understanding of magnitude by counting forward and backwards by 0.01;
- solve problems involving the multiplication and division of multi-digit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies;
- demonstrate an understanding of proportional reasoning by investigating whole-number rates.

Specific Expectations

Quantity Relationships

By the end of Grade 5, students will:

– represent, compare, and order whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);

– demonstrate an understanding of place value in whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools and strategies (e.g., use numbers to represent 23 011 as 20 000 + 3000 + 0 + 10 + 1; use base ten materials to represent the relationship between 1, 0.1, and 0.01) (*Sample problem:* How many thousands cubes would be needed to make a base ten block for 100 000?);

– read and print in words whole numbers to ten thousand, using meaningful contexts (e.g., newspapers, magazines);

– round decimal numbers to the nearest tenth, in problems arising from real-life situations;

– represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, number lines) and using standard fractional notation;

– demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that $\frac{3}{4}$ is equal to $\frac{9}{12}$);

– demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings (e.g., use base ten materials to show that three tenths [0.3] is equal to thirty hundredths [0.30]);

– read and write money amounts to $1000 (e.g., $455.35 is 455 dollars and 35 cents, or four hundred fifty-five dollars and thirty-five cents);

– solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 100 000 (*Sample problem:* How many boxes hold 100 000 sheets of paper, if one box holds 8 packages of paper, and one package of paper contains 500 sheets of paper?).

Counting

By the end of Grade 5, students will:

– count forward by hundredths from any decimal number expressed to two decimal places, using concrete materials and number lines (e.g., use base ten materials to represent 2.96 and count forward by hundredths: 2.97, 2.98, 2.99, 3.00, 3.01, …; "Two and ninety-six hundredths, two and ninety-seven hundredths, two and ninety-eight hundredths, two and ninety-nine hundredths, three, three and one hundredth, …") (*Sample problem:* What connections can you make between counting by hundredths and measuring lengths in centimetres and metres?).

Operational Sense

By the end of Grade 5, students will:

– solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies (e.g., use the commutative property: 5 x 18 x 2 = 5 x 2 x 18, which gives 10 x 18 = 180);

– add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (e.g., use 10 x 10 grids to add 2.45 and 3.25);

– multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms;

– divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms;

– multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);

– use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution (*Sample problem:* Mori used a calculator to add 7.45 and 2.39. The calculator display showed 31.35. Explain why this result is not reasonable, and suggest where you think Mori made his mistake.).

Proportional Relationships

By the end of Grade 5, students will:

– describe multiplicative relationships between quantities by using simple fractions and decimals (e.g., "If you have 4 plums and I have 6 plums, I can say that I have $1\frac{1}{2}$ or 1.5 times as many plums as you have.");

– determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms (e.g., use a 10 x 10 grid to show that $\frac{2}{5} = \frac{40}{100}$, which can also be represented as 0.4);

– demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings (*Sample problem:* If 2 books cost $6, how would you calculate the cost of 8 books?).

Grade 5: Measurement

Overall Expectations

By the end of Grade 5, students will:

- estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies;
- determine the relationships among units and measurable attributes, including the area of a rectangle and the volume of a rectangular prism.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 5, students will:

– estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest second;

– estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years (*Sample problem:* You are travelling from Toronto to Montreal by train. If the train departs Toronto at 11:30 a.m. and arrives in Montreal at 4:56 p.m., how long will you be on the train?);

– measure and record temperatures to determine and represent temperature changes over time (e.g., record temperature changes in an experiment or over a season) (*Sample problem:* Investigate the relationship between weather, climate, and temperature changes over time in different locations.);

– estimate and measure the perimeter and area of regular and irregular polygons, using a variety of tools (e.g., grid paper, geoboard, dynamic geometry software) and strategies.

Measurement Relationships

By the end of Grade 5, students will:

– select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure length, height, width, and distance, and to measure the perimeter of various polygons;

– solve problems requiring conversion from metres to centimetres and from kilometres to metres (*Sample problem:* Describe the multiplicative relationship between the number of centimetres and the number of metres that represent a length. Use this relationship to convert 5.1 m to centimetres.);

– solve problems involving the relationship between a 12-hour clock and a 24-hour clock (e.g., 15:00 is 3 hours after 12 noon, so 15:00 is the same as 3:00 p.m.);

– create, through investigation using a variety of tools (e.g., pattern blocks, geoboard, grid paper) and strategies, two-dimensional shapes with the same perimeter or the same area (e.g., rectangles and parallelograms with the same base and the same height) (*Sample problem:* Using dot paper, how many different rectangles can you draw with a perimeter of 12 units? with an area of 12 square units?);

– determine, through investigation using a variety of tools (e.g., concrete materials, dynamic geometry software, grid paper) and strategies (e.g., building arrays), the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., *Area* = *length* x *width*; *Perimeter* = (2 x *length*) + (2 x *width*)];

- solve problems requiring the estimation and calculation of perimeters and areas of rectangles (*Sample problem:* You are helping to fold towels, and you want them to stack nicely. By folding across the length and/or the width, you fold each towel a total of three times. You want the shape of each folded towel to be as close to a square as possible. Does it matter how you fold the towels?);
- determine, through investigation, the relationship between capacity (i.e., the amount a container can hold) and volume (i.e., the amount of space taken up by an object), by comparing the volume of an object with the amount of liquid it can contain or displace (e.g., a bottle has a volume, the space it takes up, and a capacity, the amount of liquid it can hold) (*Sample problem:* Compare the volume and capacity of a thin-walled container in the shape of a rectangular prism to determine the relationship between units for measuring capacity [e.g., millilitres] and units for measuring volume [e.g., cubic centimetres].);
- determine, through investigation using stacked congruent rectangular layers of concrete materials, the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula (i.e., *Volume* = *area of base* x *height*) (*Sample problem:* Create a variety of rectangular prisms using connecting cubes. For each rectangular prism, record the area of the base, the height, and the volume on a chart. Identify relationships.);
- select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram, tonne).

Grade 5: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 5, students will:

- identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures;
- identify and construct nets of prisms and pyramids;
- identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes.

Specific Expectations

Geometric Properties

By the end of Grade 5, students will:

– distinguish among polygons, regular polygons, and other two-dimensional shapes;

– distinguish among prisms, right prisms, pyramids, and other three-dimensional figures;

– identify and classify acute, right, obtuse, and straight angles;

– measure and construct angles up to 90°, using a protractor;

– identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral), and classify them according to angle and side properties;

– construct triangles, using a variety of tools (e.g., protractor, compass, dynamic geometry software), given acute or right angles and side measurements (*Sample problem:* Use a protractor, ruler, and pencil to construct a scalene triangle with a 30° angle and a side measuring 12 cm.).

Geometric Relationships

By the end of Grade 5, students will:

– identify prisms and pyramids from their nets;

– construct nets of prisms and pyramids, using a variety of tools (e.g., grid paper, isometric dot paper, Polydrons, computer application).

Location and Movement

By the end of Grade 5, students will:

– locate an object using the cardinal directions (i.e., north, south, east, west) and a coordinate system (e.g., "If I walk 5 steps north and 3 steps east, I will arrive at the apple tree.");

– compare grid systems commonly used on maps (i.e., the use of numbers and letters to identify an area; the use of a coordinate system based on the cardinal directions to describe a specific location);

– identify, perform, and describe translations, using a variety of tools (e.g., geoboard, dot paper, computer program);

– create and analyse designs by translating and/or reflecting a shape, or shapes, using a variety of tools (e.g., geoboard, grid paper, computer program) (*Sample problem:* Identify translations and/or reflections that map congruent shapes onto each other in a given design.).

Grade 5: Patterning and Algebra

Overall Expectations

By the end of Grade 5, students will:

- determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations;
- demonstrate, through investigation, an understanding of the use of variables in equations.

Specific Expectations

Patterns and Relationships

By the end of Grade 5, students will:

– create, identify, and extend numeric and geometric patterns, using a variety of tools (e.g., concrete materials, paper and pencil, calculators, spreadsheets);

– build a model to represent a number pattern presented in a table of values that shows the term number and the term;

– make a table of values for a pattern that is generated by adding or subtracting a number (i.e., a constant) to get the next term, or by multiplying or dividing by a constant to get the next term, given either the sequence (e.g., 12, 17, 22, 27, 32, …) or the pattern rule in words (e.g., start with 12 and add 5 to each term to get the next term);

– make predictions related to growing and shrinking geometric and numeric patterns (*Sample problem:* Create growing L's using tiles. The first L has 3 tiles, the second L has 5 tiles, the third L has 7 tiles, and so on. Predict the number of tiles you would need to build the 10th L in the pattern.);

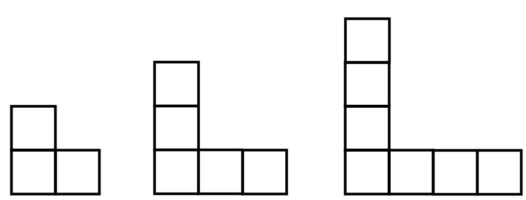


Figure 1 Figure 2 Figure 3

– extend and create repeating patterns that result from translations, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, dot paper).

Variables, Expressions, and Equations

By the end of Grade 5, students will:

– demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or other symbols that describe relationships involving simple rates (e.g., the equations $C = 3 \times n$ and $3 \times n = C$ both represent the relationship between the total cost (C), in dollars, and the number of sandwiches purchased (n), when each sandwich costs $3);

– demonstrate, through investigation, an understanding of variables as unknown quantities represented by a letter or other symbol (e.g., $12 = 5 + \square$ or $12 = 5 + s$ can be used to represent the following situation: "I have 12 stamps altogether and 5 of them are from Canada. How many are from other countries?");

– determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (*Sample problem:* What is the missing number in the equation $8 = 88 \div \square$?).

Grade 5: Data Management and Probability

Overall Expectations

By the end of Grade 5, students will:

- collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including broken-line graphs;
- read, describe, and interpret primary data and secondary data presented in charts and graphs, including broken-line graphs;
- represent as a fraction the probability that a specific outcome will occur in a simple probability experiment, using systematic lists and area models.

Specific Expectations

Collection and Organization of Data

By the end of Grade 5, students will:

– distinguish between discrete data (i.e., data organized using numbers that have gaps between them, such as whole numbers, and often used to represent a count, such as the number of times a word is used) and continuous data (i.e., data organized using all numbers on a number line that fall within the range of the data, and used to represent measurements such as heights or ages of trees);

– collect data by conducting a survey or an experiment (e.g., gather and record air temperature over a two-week period) to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;

– collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including broken-line graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales that suit the range and distribution of the data (e.g., to represent precipitation amounts ranging from 0 mm to 50 mm over the school year, use a scale of 5 mm for each unit on the vertical axis and show months on the horizontal axis), using a variety of tools (e.g., graph paper, simple spreadsheets, dynamic statistical software);

– demonstrate an understanding that sets of data can be samples of larger populations (e.g., to determine the most common shoe size in your class, you would include every member of the class in the data; to determine the most common shoe size in Ontario for your age group, you might collect a large sample from classes across the province);

– describe, through investigation, how a set of data is collected (e.g., by survey, measurement, observation) and explain whether the collection method is appropriate.

Data Relationships

By the end of Grade 5, students will:

– read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., precipitation or temperature data in the newspaper, data from the Internet about heights of buildings and other structures), presented in charts, tables, and graphs (including broken-line graphs);

– calculate the mean for a small set of data and use it to describe the shape of the data set across its range of values, using charts, tables, and graphs (e.g., "The data values fall mainly into two groups on both sides of the mean."; "The set of data is not spread out evenly around the mean.");

– compare similarities and differences between two related sets of data, using a variety of strategies (e.g., by representing the data using tally charts, stem-and-leaf plots, double bar graphs, or broken-line graphs; by determining measures of central tendency [i.e., mean, median, and mode]; by describing the shape of a data set across its range of values).

Probability

By the end of Grade 5, students will:

– determine and represent all the possible outcomes in a simple probability experiment (e.g., when tossing a coin, the possible outcomes are heads and tails; when rolling a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6), using systematic lists and area models (e.g., a rectangle is divided into two equal areas to represent the outcomes of a coin toss experiment);

– represent, using a common fraction, the probability that an event will occur in simple games and probability experiments (e.g., "My spinner has four equal sections and one of those sections is coloured red. The probability that I will land on red is $\frac{1}{4}$.");

– pose and solve simple probability problems, and solve them by conducting probability experiments and selecting appropriate methods of recording the results (e.g., tally chart, line plot, bar graph).

Grade 6

The following are highlights of student learning in Grade 6. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering numbers to 1 000 000; developing the concept of place value to thousandths; comparing and ordering fractional amounts with unlike denominators; estimating 10%, 25%, 50%, and 75% of a quantity; adding and subtracting decimal amounts to thousandths; multiplying and dividing four-digit whole numbers by two-digit whole numbers; multiplying and dividing decimals to tenths by whole numbers and two-digit by two-digit whole numbers; dividing three-digit whole numbers by one-digit whole numbers; applying order of operations in expressions without brackets; relating simple fractions, decimals, and percents

Measurement: measuring quantities using metric units; converting from larger to smaller metric units, including square metres to square centimetres; developing and applying area relationships for a parallelogram and a triangle; developing and applying the volume relationships for a triangular prism; determining and applying surface area relationships for rectangular and triangular prisms; relating square metres and square centimetres

Geometry and Spatial Sense: classifying quadrilaterals by geometric properties; sorting polygons by lines of symmetry and by rotational symmetry; measuring angles to 180° with a protractor; constructing polygons; representing figures using views and isometric sketches; performing and describing rotations; plotting points in the first quadrant

Patterning and Algebra: representing patterns using ordered pairs and graphs; describing pattern rules in words; calculating any term when given the term number; investigating variables as changing quantities; solving equations using concrete materials and guess and check

Data Management and Probability: collecting and organizing discrete and continuous data; displaying data using continuous line graphs; selecting appropriate graphical representations; using continuous line graphs and mean to compare sets of data; finding theoretical probabilities; predicting the frequency of an outcome based on the theoretical probability

Grade 6: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 6, students will:

PROBLEM SOLVING
- develop, select, and apply problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counter-examples) to make and investigate conjectures and construct and defend arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., by using physical models, pictures, numbers, variables, diagrams, graphs, onscreen dynamic representations), make connections among them, and apply them to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, and observing basic mathematical conventions.

Grade 6: Number Sense and Numeration

Overall Expectations

By the end of Grade 6, students will:

- read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
- demonstrate an understanding of relationships involving percent, ratio, and unit rate.

Specific Expectations

Quantity Relationships

By the end of Grade 6, students will:

– represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);

– demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools and strategies (e.g. use base ten materials to represent the relationship between 1, 0.1, 0.01, and 0.001) (*Sample problem:* How many thousands cubes would be needed to make a base ten block for 1 000 000?);

– read and print in words whole numbers to one hundred thousand, using meaningful contexts (e.g., the Internet, reference books);

– represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation (*Sample problem:* Use fraction strips to show that $1\frac{1}{2}$ is greater than $\frac{5}{4}$.);

– estimate quantities using benchmarks of 10%, 25%, 50%, 75%, and 100% (e.g., the container is about 75% full; approximately 50% of our students walk to school);

– solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1 000 000 (*Sample problem:* How would you determine if a person could live to be 1 000 000 hours old? Show your work.);

– identify composite numbers and prime numbers, and explain the relationship between them (i.e., any composite number can be factored into prime factors) (e.g., 42 = 2 x 3 x 7).

Operational Sense

By the end of Grade 6, students will:

– use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers (e.g., use the commutative property: 4 x 16 x 5 = 4 x 5 x 16, which gives 20 x 16 = 320; use the distributive property: (500 + 15) ÷ 5 = 500 ÷ 5 + 15 ÷ 5, which gives 100 + 3 = 103);

– solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);

– add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators;
– multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (e.g., calculate 4 x 1.4 using base ten materials; calculate 5.6 ÷ 4 using base ten materials);
– multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);
– multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies (e.g., "To convert 0.6 m^2 to square centimetres, I calculated in my head 0.6 x 10 000 and got 6000 cm^2.") (*Sample problem:* Use a calculator to help you generalize a rule for multiplying numbers by 10 000.);
– use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution;
– explain the need for a standard order for performing operations, by investigating the impact that changing the order has when performing a series of operations (*Sample problem:* Calculate and compare the answers to 3 + 2 x 5 using a basic four-function calculator and using a scientific calculator.).

Proportional Relationships

By the end of Grade 6, students will:

– represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation (*Sample problem:* In a classroom of 28 students, 12 are female. What is the ratio of male students to female students?);
– determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a 10 x 10 grid to show that $\frac{1}{4}$ = 0.25 or 25%);
– represent relationships using unit rates (*Sample problem:* If 5 batteries cost $4.75, what is the cost of 1 battery?).

Grade 6: Measurement

Overall Expectations

By the end of Grade 6, students will:

- estimate, measure, and record quantities, using the metric measurement system;
- determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 6, students will:

– demonstrate an understanding of the relationship between estimated and precise measurements, and determine and justify when each kind is appropriate (*Sample problem:* You are asked how long it takes you to travel a given distance. How is the method you use to determine the time related to the precision of the measurement?);

– estimate, measure, and record length, area, mass, capacity, and volume, using the metric measurement system.

Measurement Relationships

By the end of Grade 6, students will:

– select and justify the appropriate metric unit (i.e., millimetre, centimetre, decimetre, metre, decametre, kilometre) to measure length or distance in a given real-life situation (*Sample problem:* Select and justify the unit that should be used to measure the perimeter of the school.);

– solve problems requiring conversion from larger to smaller metric units (e.g., metres to centimetres, kilograms to grams, litres to millilitres) (*Sample problem:* How many grams are in one serving if 1.5 kg will serve six people?);

– construct a rectangle, a square, a triangle, and a parallelogram, using a variety of tools (e.g., concrete materials, geoboard, dynamic geometry software, grid paper), given the area and/or perimeter (*Sample problem:* Create two different triangles with an area of 12 square units, using a geoboard.);

– determine, through investigation using a variety of tools (e.g., pattern blocks, Power Polygons, dynamic geometry software, grid paper) and strategies (e.g., paper folding, cutting, and rearranging), the relationship between the area of a rectangle and the areas of parallelograms and triangles, by decomposing (e.g., cutting up a parallelogram into a rectangle and two congruent triangles) and composing (e.g., combining two congruent triangles to form a parallelogram) (*Sample problem:* Decompose a rectangle and rearrange the parts to compose a parallelogram with the same area. Decompose a parallelogram into two congruent triangles, and compare the area of one of the triangles with the area of the parallelogram.);

– develop the formulas for the area of a parallelogram (i.e., *Area of parallelogram* = *base* x *height*) and the area of a triangle [i.e., *Area of triangle* = (*base* x *height*) ÷ 2], using the area relationships among rectangles, parallelograms, and triangles (*Sample problem:* Use dynamic geometry software to show that parallelograms with the same height and the same base all have the same area.);

– solve problems involving the estimation and calculation of the areas of triangles and the areas of parallelograms (*Sample problem:* Calculate the areas of parallelograms that share the same base and the same height, including the special case where the parallelogram is a rectangle.);
– determine, using concrete materials, the relationship between units used to measure area (i.e., square centimetre, square metre), and apply the relationship to solve problems that involve conversions from square metres to square centimetres (*Sample problem:* Describe the multiplicative relationship between the number of square centimetres and the number of square metres that represent an area. Use this relationship to determine how many square centimetres fit into half a square metre.);
– determine, through investigation using a variety of tools and strategies (e.g., decomposing rectangular prisms into triangular prisms; stacking congruent triangular layers of concrete materials to form a triangular prism), the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula (i.e., *Volume = area of base* x *height*) (*Sample problem:* Create triangular prisms by splitting rectangular prisms in half. For each prism, record the area of the base, the height, and the volume on a chart. Identify relationships.);
– determine, through investigation using a variety of tools (e.g., nets, concrete materials, dynamic geometry software, Polydrons) and strategies, the surface area of rectangular and triangular prisms;
– solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms (*Sample problem:* How many square centimetres of wrapping paper are required to wrap a box that is 10 cm long, 8 cm wide, and 12 cm high?).

Grade 6: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 6, students will:

- classify and construct polygons and angles;
- sketch three-dimensional figures, and construct three-dimensional figures from drawings;
- describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes.

Specific Expectations

Geometric Properties

By the end of Grade 6, students will:

– sort and classify quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools (e.g., geoboard, dynamic geometry software) and strategies (e.g., using charts, using Venn diagrams);

– sort polygons according to the number of lines of symmetry and the order of rotational symmetry, through investigation using a variety of tools (e.g., tracing paper, dynamic geometry software, Mira);

– measure and construct angles up to 180° using a protractor, and classify them as acute, right, obtuse, or straight angles;

– construct polygons using a variety of tools, given angle and side measurements (*Sample problem:* Use dynamic geometry software to construct trapezoids with a 45° angle and a side measuring 11 cm.).

Geometric Relationships

By the end of Grade 6, students will:

– build three-dimensional models using connecting cubes, given isometric sketches or different views (i.e., top, side, front) of the structure (*Sample problem:* Given the top, side, and front views of a structure, build it using the smallest number of cubes possible.);

– sketch, using a variety of tools (e.g., isometric dot paper, dynamic geometry software), isometric perspectives and different views (i.e., top, side, front) of three-dimensional figures built with interlocking cubes.

Location and Movement

By the end of Grade 6, students will:

– explain how a coordinate system represents location, and plot points in the first quadrant of a Cartesian coordinate plane;

– identify, perform, and describe, through investigation using a variety of tools (e.g., grid paper, tissue paper, protractor, computer technology), rotations of 180º and clockwise and counterclockwise rotations of 90°, with the centre of rotation inside or outside the shape;

– create and analyse designs made by reflecting, translating, and/or rotating a shape, or shapes, by 90º or 180º (*Sample problem:* Identify rotations of 90° or 180° that map congruent shapes, in a given design, onto each other.).

Grade 6: Patterning and Algebra

Overall Expectations

By the end of Grade 6, students will:

- describe and represent relationships in growing and shrinking patterns (where the terms are whole numbers), and investigate repeating patterns involving rotations;
- use variables in simple algebraic expressions and equations to describe relationships.

Specific Expectations

Patterns and Relationships

By the end of Grade 6, students will:

– identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically;

– make tables of values, for growing patterns given pattern rules, in words (e.g., start with 3, then double each term and add 1 to get the next term), then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a variety of tools (e.g., graph paper, calculators, dynamic statistical software);

– determine the term number of a given term in a growing pattern that is represented by a pattern rule in words, a table of values, or a graph (*Sample problem:* For the pattern rule "start with 1 and add 3 to each term to get the next term", use graphing to find the term number when the term is 19.);

– describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (e.g., for 1, 3, 5, 7, 9, …, the pattern rule is "start with 1 and add 2 to each term to get the next term"), then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number (e.g., for 2, 4, 6, 8, …, the pattern rule for the general term is "double the term number");

– determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (*Sample problem:* For the pattern 5000, 4750, 4500, 4250, 4000, 3750, …, find the 15th term. Explain your reasoning.);

– extend and create repeating patterns that result from rotations, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, geoboards, dot paper).

Variables, Expressions, and Equations

By the end of Grade 6, students will:

– demonstrate an understanding of different ways in which variables are used (e.g., variable as an unknown quantity; variable as a changing quantity);

– identify, through investigation, the quantities in an equation that vary and those that remain constant (e.g., in the formula for the area of a triangle, $A = \frac{b \times h}{2}$, the number 2 is a constant, whereas b and h can vary and may change the value of A);

– solve problems that use two or three symbols or letters as variables to represent different unknown quantities (*Sample problem:* If $n + l = 15$ and $n + l + s = 19$, what value does the s represent?);

– determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (***Sample problem:*** Use the method of your choice to determine the value of the variable in the equation $2 \times n + 3 = 11$. Is there more than one possible solution? Explain your reasoning.).

Grade 6: Data Management and Probability

Overall Expectations

By the end of Grade 6, students will:

- collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including continuous line graphs;
- read, describe, and interpret data, and explain relationships between sets of data;
- determine the theoretical probability of an outcome in a probability experiment, and use it to predict the frequency of the outcome.

Specific Expectations

Collection and Organization of Data

By the end of Grade 6, students will:

– collect data by conducting a survey (e.g., use an Internet survey tool) or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;

– collect and organize discrete or continuous primary data and secondary data (e.g., electronic data from websites such as E-Stat or Census At Schools) and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, spreadsheets, dynamic statistical software);

– select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied, such as pictographs, horizontal or vertical bar graphs, stem-and-leaf plots, double bar graphs, broken-line graphs, and continuous line graphs);

– determine, through investigation, how well a set of data represents a population, on the basis of the method that was used to collect the data (*Sample problem:* Would the results of a survey of primary students about their favourite television shows represent the favourite shows of students in the entire school? Why or why not?).

Data Relationships

By the end of Grade 6, students will:

– read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., sports data in the newspaper, data from the Internet about movies), presented in charts, tables, and graphs (including continuous line graphs);

– compare, through investigation, different graphical representations of the same data (*Sample problem:* Use technology to help you compare the different types of graphs that can be created to represent a set of data about the number of runs or goals scored against each team in a tournament. Describe the similarities and differences that you observe.);

– explain how different scales used on graphs can influence conclusions drawn from the data;

– demonstrate an understanding of mean (e.g., *mean* differs from *median* and *mode* because it is a value that "balances" a set of data – like the centre point or fulcrum in a lever), and use the mean to compare two sets of related data, with and without the use of technology (*Sample problem:* Use the mean to compare the masses of backpacks of students from two or more Grade 6 classes.);

– demonstrate, through investigation, an understanding of how data from charts, tables, and graphs can be used to make inferences and convincing arguments (e.g., describe examples found in newspapers and magazines).

Probability

By the end of Grade 6, students will:

– express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely (e.g., the theoretical probability of rolling an odd number on a six-sided number cube is $\frac{3}{6}$ because, of six equally likely outcomes, only three are favourable – that is, the odd numbers 1, 3, 5);

– represent the probability of an event (i.e., the likelihood that the event will occur), using a value from the range of 0 (never happens or impossible) to 1 (always happens or certain);

– predict the frequency of an outcome of a simple probability experiment or game, by calculating and using the theoretical probability of that outcome (e.g., "The theoretical probability of spinning red is $\frac{1}{4}$ since there are four different-coloured areas that are equal. If I spin my spinner 100 times, I predict that red should come up about 25 times."). (*Sample problem:* Create a spinner that has rotational symmetry. Predict how often the spinner will land on the same sector after 25 spins. Perform the experiment and compare the prediction to the results.).

Grade 7

The following are highlights of student learning in Grade 7. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering decimals (to hundredths), fractions, and integers; representing squares and square roots; dividing whole numbers by simple fractions and decimals; adding and subtracting simple fractions and integers; multiplying and dividing decimal numbers to thousandths by one-digit whole numbers; applying order of operations in expressions with brackets; relating fractions, decimals, and percents; solving problems involving whole-number percents and unit rates

Measurement: converting between metric units, including converting between square centimetres and square metres; developing the area relationship for a trapezoid; developing and applying the formula for the volume of a prism; determining and applying surface-area relationships for prisms; relating millilitres and cubic centimetres

Geometry and Spatial Sense: constructing parallel, perpendicular, and intersecting lines; sorting and classifying triangles and quadrilaterals by geometric properties; constructing angle bisectors and perpendicular bisectors; investigating relationships among congruent shapes; relating enlarging and reducing to similar shapes; comparing similar and congruent shapes; performing and describing dilatations; tiling a plane; plotting points in all four quadrants

Patterning and Algebra: representing linear growing patterns; representing patterns algebraically; modelling real-life relationships involving constant rates graphically and algebraically; translating phrases, using algebraic expressions; finding the term in a pattern algebraically when given any term number; solving linear equations using concrete materials or inspection and guess and check

Data Management and Probability: collecting and organizing categorical, discrete, and continuous data; displaying data in relative frequency tables and circle graphs; identifying bias in data; relating changes in data to changes in central tendency; making inferences based on data; investigating real-world applications of probability; determining the theoretical probability of two independent events

Grade 7: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 7, students will:

PROBLEM SOLVING
- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Grade 7: Number Sense and Numeration

Overall Expectations

By the end of Grade 7, students will:

- represent, compare, and order numbers, including integers;
- demonstrate an understanding of addition and subtraction of fractions and integers, and apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers;
- demonstrate an understanding of proportional relationships using percent, ratio, and rate.

Specific Expectations

Quantity Relationships

By the end of Grade 7, students will:

– represent, compare, and order decimals to hundredths and fractions, using a variety of tools (e.g., number lines, Cuisenaire rods, base ten materials, calculators);

– generate multiples and factors, using a variety of tools and strategies (e.g., identify multiples on a hundreds chart; create rectangles on a geoboard) (*Sample problem:* List all the rectangles that have an area of 36 cm^2 and have whole-number dimensions.);

– identify and compare integers found in real-life contexts (e.g., –10°C is much colder than +5°C);

– represent and order integers, using a variety of tools (e.g., two-colour counters, virtual manipulatives, number lines);

– select and justify the most appropriate representation of a quantity (i.e., fraction, decimal, percent) for a given context (e.g., "I would use a decimal for recording the length or mass of an object, and a fraction for part of an hour.");

– represent perfect squares and square roots, using a variety of tools (e.g., geoboards, connecting cubes, grid paper);

– explain the relationship between exponential notation and the measurement of area and volume (*Sample problem:* Explain why area is expressed in square units [$units^2$] and volume is expressed in cubic units [$units^3$].).

Operational Sense

By the end of Grade 7, students will:

– divide whole numbers by simple fractions and by decimal numbers to hundredths, using concrete materials (e.g., divide 3 by $\frac{1}{2}$ using fraction strips; divide 4 by 0.8 using base ten materials and estimation);

– use a variety of mental strategies to solve problems involving the addition and subtraction of fractions and decimals (e.g., use the commutative property: $3 \times \frac{2}{5} \times \frac{1}{3} = 3 \times \frac{1}{3} \times \frac{2}{5}$, which gives $1 \times \frac{2}{5} = \frac{2}{5}$; use the distributive property: $16.8 \div 0.2$ can be thought of as $(16 + 0.8) \div 0.2 = 16 \div 0.2 + 0.8 \div 0.2$, which gives $80 + 4 = 84$);

– solve problems involving the multiplication and division of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);

- solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);
- use estimation when solving problems involving operations with whole numbers, decimals, and percents, to help judge the reasonableness of a solution (*Sample problem:* A book costs $18.49. The salesperson tells you that the total price, including taxes, is $22.37. How can you tell if the total price is reasonable without using a calculator?);
- evaluate expressions that involve whole numbers and decimals, including expressions that contain brackets, using order of operations;
- add and subtract fractions with simple like and unlike denominators, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, calculators) and algorithms;
- demonstrate, using concrete materials, the relationship between the repeated addition of fractions and the multiplication of that fraction by a whole number (e.g., $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \times \frac{1}{2}$);
- add and subtract integers, using a variety of tools (e.g., two-colour counters, virtual manipulatives, number lines).

Proportional Relationships

By the end of Grade 7, students will:

- determine, through investigation, the relationships among fractions, decimals, percents, and ratios;
- solve problems that involve determining whole number percents, using a variety of tools (e.g., base ten materials, paper and pencil, calculators) (*Sample problem:* If there are 5 blue marbles in a bag of 20 marbles, what percent of the marbles are not blue?);
- demonstrate an understanding of rate as a comparison, or ratio, of two measurements with different units (e.g., speed is a rate that compares distance to time and that can be expressed as kilometres per hour);
- solve problems involving the calculation of unit rates (*Sample problem:* You go shopping and notice that 25 kg of Ryan's Famous Potatoes cost $12.95, and 10 kg of Gillian's Potatoes cost $5.78. Which is the better deal? Justify your answer.).

Grade 7: Measurement

Overall Expectations

By the end of Grade 7, students will:

- report on research into real-life applications of area measurements;
- determine the relationships among units and measurable attributes, including the area of a trapezoid and the volume of a right prism.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 7, students will:

– research and report on real-life applications of area measurements (e.g., building a skateboard; painting a room).

Measurement Relationships

By the end of Grade 7, students will:

– sketch different polygonal prisms that share the same volume (*Sample problem:* The Neuman Company is designing a new container for its marbles. The container must have a volume of 200 cm^3. Sketch three possible containers, and explain which one you would recommend.);

– solve problems that require conversion between metric units of measure (e.g., millimetres and centimetres, grams and kilograms, millilitres and litres) (*Sample problem:* At Andrew's Deli, cheese is on sale for $11.50 for one kilogram. How much would it cost to purchase 150 g of cheese?);

– solve problems that require conversion between metric units of area (i.e., square centimetres, square metres) (*Sample problem:* What is the ratio of the number of square metres to the number of square centimetres for a given area? Use this ratio to convert 6.25 m^2 to square centimetres.);

– determine, through investigation using a variety of tools (e.g., concrete materials, dynamic geometry software) and strategies, the relationship for calculating the area of a trapezoid, and generalize to develop the formula [i.e., *Area* = (*sum of lengths of parallel sides* x *height*) ÷ 2] (*Sample problem:* Determine the relationship between the area of a parallelogram and the area of a trapezoid by composing a parallelogram from congruent trapezoids.);

– solve problems involving the estimation and calculation of the area of a trapezoid;

– estimate and calculate the area of composite two-dimensional shapes by decomposing into shapes with known area relationships (e.g., rectangle, parallelogram, triangle) (*Sample problem:* Decompose a pentagon into shapes with known area relationships to find the area of the pentagon.);

– determine, through investigation using a variety of tools and strategies (e.g., decomposing right prisms; stacking congruent layers of concrete materials to form a right prism), the relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases (e.g., parallelograms, trapezoids), and generalize to develop the formula (i.e., *Volume* = *area of base* x *height*) (*Sample problem:* Decompose right prisms with simple polygonal bases into triangular prisms and rectangular prisms. For each prism, record the area of the base, the height, and the volume on a chart. Identify relationships.);

– determine, through investigation using a variety of tools (e.g., nets, concrete materials, dynamic geometry software, Polydrons), the surface area of right prisms;
– solve problems that involve the surface area and volume of right prisms and that require conversion between metric measures of capacity and volume (i.e., millilitres and cubic centimetres) (***Sample problem:*** An aquarium has a base in the shape of a trapezoid. The aquarium is 75 cm high. The base is 50 cm long at the front, 75 cm long at the back, and 25 cm wide. Find the capacity of the aquarium.).

Grade 7: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 7, students will:

- construct related lines, and classify triangles, quadrilaterals, and prisms;
- develop an understanding of similarity, and distinguish similarity and congruence;
- describe location in the four quadrants of a coordinate system, dilatate two-dimensional shapes, and apply transformations to create and analyse designs.

Specific Expectations

Geometric Properties

By the end of Grade 7, students will:

– construct related lines (i.e., parallel; perpendicular; intersecting at 30º, 45º, and 60º), using angle properties and a variety of tools (e.g., compass and straight edge, protractor, dynamic geometry software) and strategies (e.g., paper folding);

– sort and classify triangles and quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools (e.g., geoboard, dynamic geometry software) and strategies (e.g., using charts, using Venn diagrams) (*Sample problem:* Investigate whether dilatations change the geometric properties of triangles and quadrilaterals.);

– construct angle bisectors and perpendicular bisectors, using a variety of tools (e.g., Mira, dynamic geometry software, compass) and strategies (e.g., paper folding), and represent equal angles and equal lengths using mathematical notation;

– investigate, using concrete materials, the angles between the faces of a prism, and identify right prisms (*Sample problem:* Identify the perpendicular faces in a set of right prisms.).

Geometric Relationships

By the end of Grade 7, students will:

– identify, through investigation, the minimum side and angle information (i.e., side-side-side; side-angle-side; angle-side-angle) needed to describe a unique triangle (e.g., "I can draw many triangles if I'm only told the length of one side, but there's only one triangle I can draw if you tell me the lengths of all three sides.");

– determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials, geoboard), relationships among area, perimeter, corresponding side lengths, and corresponding angles of congruent shapes (*Sample problem:* Do you agree with the conjecture that triangles with the same area must be congruent? Justify your reasoning.);

– demonstrate an understanding that enlarging or reducing two-dimensional shapes creates similar shapes;

– distinguish between and compare similar shapes and congruent shapes, using a variety of tools (e.g., pattern blocks, grid paper, dynamic geometry software) and strategies (e.g., by showing that dilatations create similar shapes and that translations, rotations, and reflections generate congruent shapes) (*Sample problem:* A larger square can be composed from four congruent square pattern blocks. Identify another pattern block you can use to compose a larger shape that is similar to the shape of the block.).

Location and Movement

By the end of Grade 7, students will:

– plot points using all four quadrants of the Cartesian coordinate plane;

– identify, perform, and describe dilatations (i.e., enlargements and reductions), through investigation using a variety of tools (e.g., dynamic geometry software, geoboard, pattern blocks, grid paper);

– create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools (e.g., concrete materials, Mira, drawings, dynamic geometry software) and strategies (e.g., paper folding) (*Sample problem:* Identify transformations that may be observed in architecture or in artwork [e.g., in the art of M.C. Escher].);

– determine, through investigation using a variety of tools (e.g., pattern blocks, Polydrons, grid paper, tiling software, dynamic geometry software, concrete materials), polygons or combinations of polygons that tile a plane, and describe the transformation(s) involved.

Grade 7: Patterning and Algebra

Overall Expectations

By the end of Grade 7, students will:

- represent linear growing patterns (where the terms are whole numbers) using concrete materials, graphs, and algebraic expressions;
- model real-life linear relationships graphically and algebraically, and solve simple algebraic equations using a variety of strategies, including inspection and guess and check.

Specific Expectations

Patterns and Relationships

By the end of Grade 7, students will:

– represent linear growing patterns, using a variety of tools (e.g., concrete materials, paper and pencil, calculators, spreadsheets) and strategies (e.g., make a table of values using the term number and the term; plot the coordinates on a graph; write a pattern rule using words);

– make predictions about linear growing patterns, through investigation with concrete materials (*Sample problem:* Investigate the surface area of towers made from a single column of connecting cubes, and predict the surface area of a tower that is 50 cubes high. Explain your reasoning.);

– develop and represent the general term of a linear growing pattern, using algebraic expressions involving one operation (e.g., the general term for the sequence 4, 5, 6, 7, … can be written algebraically as $n + 3$, where n represents the term number; the general term for the sequence 5, 10, 15, 20, … can be written algebraically as $5n$, where n represents the term number);

– compare pattern rules that generate a pattern by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (e.g., for 1, 3, 5, 7, 9, …, the pattern rule is "start at 1 and add 2 to each term to get the next term") with pattern rules that use the term number to describe the general term (e.g., for 1, 3, 5, 7, 9, …, the pattern rule is "double the term number and subtract 1", which can be written algebraically as $2 \times n - 1$) (*Sample problem:* For the pattern 1, 3, 5, 7, 9, …, investigate and compare different ways of finding the 50th term.).

Variables, Expressions, and Equations

By the end of Grade 7, students will:

– model real-life relationships involving constant rates where the initial condition starts at 0 (e.g., speed, heart rate, billing rate), through investigation using tables of values and graphs (*Sample problem:* Create a table of values and graph the relationship between distance and time for a car travelling at a constant speed of 40 km/h. At that speed, how far would the car travel in 3.5 h? How many hours would it take to travel 220 km?);

– model real-life relationships involving constant rates (e.g., speed, heart rate, billing rate), using algebraic equations with variables to represent the changing quantities in the relationship (e.g., the equation $p = 4t$ represents the relationship between the total number of people that can be seated (p) and the number of tables (t), given that each table can seat 4 people [4 people per table is the constant rate]);

- translate phrases describing simple mathematical relationships into algebraic expressions (e.g., one more than three times a number can be written algebraically as $1 + 3x$ or $3x + 1$), using concrete materials (e.g., algebra tiles, pattern blocks, counters);
- evaluate algebraic expressions by substituting natural numbers for the variables;
- make connections between evaluating algebraic expressions and determining the term in a pattern using the general term (e.g., for 3, 5, 7, 9, …, the general term is the algebraic expression $2n + 1$; evaluating this expression when $n = 12$ tells you that the 12th term is $2(12) + 1$, which equals 25);
- solve linear equations of the form $ax = c$ or $c = ax$ and $ax + b = c$ or variations such as $b + ax = c$ and $c = bx + a$ (where a, b, and c are natural numbers) by modelling with concrete materials, by inspection, or by guess and check, with and without the aid of a calculator (e.g., "I solved $x + 7 = 15$ by using guess and check. First I tried 6 for x. Since I knew that 6 plus 7 equals 13 and 13, is less than 15, then I knew that x must be greater than 6.").

Grade 7: Data Management and Probability

Overall Expectations

By the end of Grade 7, students will:

- collect and organize categorical, discrete, or continuous primary data and secondary data and display the data using charts and graphs, including relative frequency tables and circle graphs;
- make and evaluate convincing arguments, based on the analysis of data;
- compare experimental probabilities with the theoretical probability of an outcome involving two independent events.

Specific Expectations

Collection and Organization of Data

By the end of Grade 7, students will:

- collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject and record observations or measurements;
- collect and organize categorical, discrete, or continuous primary data and secondary data (e.g., electronic data from websites such as E-Stat or Census At Schools) and display the data in charts, tables, and graphs (including relative frequency tables and circle graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, spreadsheets, dynamic statistical software);
- select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied);
- distinguish between a census and a sample from a population;
- identify bias in data collection methods (*Sample problem:* How reliable are your results if you only sample girls to determine the favourite type of book read by students in your grade?).

Data Relationships

By the end of Grade 7, students will:

- read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., temperature data or community data in the newspaper, data from the Internet about populations) presented in charts, tables, and graphs (including relative frequency tables and circle graphs);
- identify, through investigation, graphs that present data in misleading ways (e.g., line graphs that exaggerate change by starting the vertical axis at a point greater than zero);
- determine, through investigation, the effect on a measure of central tendency (i.e., mean, median, and mode) of adding or removing a value or values (e.g., changing the value of an outlier may have a significant effect on the mean but no effect on the median) (*Sample problem:* Use a set of data whose distribution across its range looks symmetrical, and change some of the values so that the distribution no longer looks symmetrical. Does the change affect the median more than the mean? Explain your thinking.);
- identify and describe trends, based on the distribution of the data presented in tables and graphs, using informal language;

– make inferences and convincing arguments that are based on the analysis of charts, tables, and graphs (*Sample problem:* Use census information to predict whether Canada's population is likely to increase.).

Probability

By the end of Grade 7, students will:

– research and report on real-world applications of probabilities expressed in fraction, decimal, and percent form (e.g., lotteries, batting averages, weather forecasts, elections);

– make predictions about a population when given a probability (*Sample problem:* The probability that a fish caught in Lake Goodfish is a bass is 29%. Predict how many bass will be caught in a fishing derby there, if 500 fish are caught.);

– represent in a variety of ways (e.g., tree diagrams, tables, models, systematic lists) all the possible outcomes of a probability experiment involving two independent events (i.e., one event does not affect the other event), and determine the theoretical probability of a specific outcome involving two independent events (*Sample problem:* What is the probability of rolling a 4 and spinning red, when you roll a number cube and spin a spinner that is equally divided into four different colours?);

– perform a simple probability experiment involving two independent events, and compare the experimental probability with the theoretical probability of a specific outcome (*Sample problem:* Place 1 red counter and 1 blue counter in an opaque bag. Draw a counter, replace it, shake the bag, and draw again. Compare the theoretical and experimental probabilities of drawing a red counter 2 times in a row.).

Grade 8

The following are highlights of student learning in Grade 8. They are provided to give teachers and parents a quick overview of the mathematical knowledge and skills that students are expected to acquire in each strand in this grade. The expectations on the pages that follow outline the required knowledge and skills in detail and provide information about the ways in which students are expected to demonstrate their learning, how deeply they will explore concepts and at what level of complexity they will perform procedures, and the mathematical processes they will learn and apply throughout the grade.

Number Sense and Numeration: representing and ordering rational numbers; representing numbers using exponential notation; solving multi-step problems involving whole numbers and decimals; multiplying and dividing fractions and integers; multiplying and dividing decimals by powers of ten; applying order of operations in expressions with brackets and exponents; solving problems involving percents to one decimal place and percents greater than 100; solving problems involving rates and proportions

Measurement: converting between cubic centimetres and cubic metres and between millilitres and cubic centimetres; developing circumference and area relationships for a circle; developing and applying the formula for the volume of a cylinder; determining and applying surface-area relationships for cylinders

Geometry and Spatial Sense: sorting quadrilaterals by geometric properties involving diagonals; constructing circles; investigating relationships among similar shapes; determining and applying angle relationships for parallel and intersecting lines; relating the numbers of faces, edges, and vertices of a polyhedron; determining and applying the Pythagorean relationship geometrically; plotting the image of a point on the coordinate plane after applying a transformation

Patterning and Algebra: representing the general term in a linear sequence, using one or more algebraic expressions; translating statements, using algebraic equations; finding the term number in a pattern algebraically when given any term; solving linear equations involving one-variable terms with integer solutions using a "balance" model

Data Management and Probability: collecting categorical, discrete, and continuous data; organizing data into intervals; displaying data using histograms and scatter plots; using measures of central tendency to compare sets of data; comparing two attributes using data management tools; comparing experimental and theoretical probabilities; calculating the probability of complementary events

Grade 8: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

Throughout Grade 8, students will:

PROBLEM SOLVING
- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Grade 8: Number Sense and Numeration

Overall Expectations

By the end of Grade 8, students will:

- represent, compare, and order equivalent representations of numbers, including those involving positive exponents;
- solve problems involving whole numbers, decimal numbers, fractions, and integers, using a variety of computational strategies;
- solve problems by using proportional reasoning in a variety of meaningful contexts.

Specific Expectations

Quantity Relationships

By the end of Grade 8, students will:

– express repeated multiplication using exponential notation (e.g., $2 \times 2 \times 2 \times 2 = 2^4$);

– represent whole numbers in expanded form using powers of ten (e.g., $347 = 3 \times 10^2 + 4 \times 10^1 + 7$);

– represent, compare, and order rational numbers (i.e., positive and negative fractions and decimals to thousandths);

– translate between equivalent forms of a number (i.e., decimals, fractions, percents) (e.g., $\frac{3}{4} = 0.75$);

– determine common factors and common multiples using the prime factorization of numbers (e.g., the prime factorization of 12 is 2 x 2 x 3; the prime factorization of 18 is 2 x 3 x 3; the greatest common factor of 12 and 18 is 2 x 3 or 6; the least common multiple of 12 and 18 is 2 x 2 x 3 x 3 or 36).

Operational Sense

By the end of Grade 8, students will:

– solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools (e.g., graphs, calculators) and strategies (e.g., estimation, algorithms);

– solve problems involving percents expressed to one decimal place (e.g., 12.5%) and whole-number percents greater than 100 (e.g., 115%) (*Sample problem:* The total cost of an item with tax included [115%] is $23.00. Use base ten materials to determine the price before tax.);

– use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution;

– represent the multiplication and division of fractions, using a variety of tools and strategies (e.g., use an area model to represent $\frac{1}{4}$ multiplied by $\frac{1}{3}$);

– solve problems involving addition, subtraction, multiplication, and division with simple fractions;

– represent the multiplication and division of integers, using a variety of tools [e.g., if black counters represent positive amounts and red counters represent negative amounts, you can model 3 x (–2) as three groups of two red counters];

– solve problems involving operations with integers, using a variety of tools (e.g., two-colour counters, virtual manipulatives, number lines);

– evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations;
– multiply and divide decimal numbers by various powers of ten (e.g., "To convert 230 000 cm^3 to cubic metres, I calculated in my head 230 000 ÷ 10^6 to get 0.23 m^3.") (*Sample problem:* Use a calculator to help you generalize a rule for dividing numbers by 1 000 000.);
– estimate, and verify using a calculator, the positive square roots of whole numbers, and distinguish between whole numbers that have whole-number square roots (i.e., perfect square numbers) and those that do not (*Sample problem:* Explain why a square with an area of 20 cm^2 does not have a whole-number side length.).

Proportional Relationships

By the end of Grade 8, students will:

– identify and describe real-life situations involving two quantities that are directly proportional (e.g., the number of servings and the quantities in a recipe, mass and volume of a substance, circumference and diameter of a circle);
– solve problems involving proportions, using concrete materials, drawings, and variables (*Sample problem:* The ratio of stone to sand in HardFast Concrete is 2 to 3. How much stone is needed if 15 bags of sand are used?);
– solve problems involving percent that arise from real-life contexts (e.g., discount, sales tax, simple interest) (*Sample problem:* In Ontario, people often pay a provincial sales tax [PST] of 8% and a federal sales tax [GST] of 7% when they make a purchase. Does it matter which tax is calculated first? Explain your reasoning.);
– solve problems involving rates (*Sample problem:* A pack of 24 CDs costs $7.99. A pack of 50 CDs costs $10.45. What is the most economical way to purchase 130 CDs?).

Grade 8: Measurement

Overall Expectations

By the end of Grade 8, students will:

- research, describe, and report on applications of volume and capacity measurement;
- determine the relationships among units and measurable attributes, including the area of a circle and the volume of a cylinder.

Specific Expectations

Attributes, Units, and Measurement Sense

By the end of Grade 8, students will:

– research, describe, and report on applications of volume and capacity measurement (e.g., cooking, closet space, aquarium size) (*Sample problem:* Describe situations where volume and capacity are used in your home.).

Measurement Relationships

By the end of Grade 8, students will:

– solve problems that require conversions involving metric units of area, volume, and capacity (i.e., square centimetres and square metres; cubic centimetres and cubic metres; millilitres and cubic centimetres) (*Sample problem:* What is the capacity of a cylindrical beaker with a radius of 5 cm and a height of 15 cm?);

– measure the circumference, radius, and diameter of circular objects, using concrete materials (*Sample Problem:* Use string to measure the circumferences of different circular objects.);

– determine, through investigation using a variety of tools (e.g., cans and string, dynamic geometry software) and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas [i.e., *Circumference of a circle* = π x *diameter*; *Area of a circle* = π x (*radius*)^2] (*Sample problem:* Use string to measure the circumferences and the diameters of a variety of cylindrical cans, and investigate the ratio of the circumference to the diameter.);

– solve problems involving the estimation and calculation of the circumference and the area of a circle;

– determine, through investigation using a variety of tools and strategies (e.g., generalizing from the volume relationship for right prisms, and verifying using the capacity of thin-walled cylindrical containers), the relationship between the area of the base and height and the volume of a cylinder, and generalize to develop the formula (i.e., *Volume* = *area of base* x *height*);

– determine, through investigation using concrete materials, the surface area of a cylinder (*Sample problem:* Use the label and the plastic lid from a cylindrical container to help determine its surface area.);

– solve problems involving the surface area and the volume of cylinders, using a variety of strategies (*Sample problem:* Compare the volumes of the two cylinders that can be created by taping the top and bottom, or the other two sides, of a standard sheet of paper.).

Grade 8: Geometry and Spatial Sense

Overall Expectations

By the end of Grade 8, students will:

- demonstrate an understanding of the geometric properties of quadrilaterals and circles and the applications of geometric properties in the real world;
- develop geometric relationships involving lines, triangles, and polyhedra, and solve problems involving lines and triangles;
- represent transformations using the Cartesian coordinate plane, and make connections between transformations and the real world.

Specific Expectations

Geometric Properties

By the end of Grade 8, students will:

– sort and classify quadrilaterals by geometric properties, including those based on diagonals, through investigation using a variety of tools (e.g., concrete materials, dynamic geometry software) (*Sample problem:* Which quadrilaterals have diagonals that bisect each other perpendicularly?);

– construct a circle, given its centre and radius, or its centre and a point on the circle, or three points on the circle;

– investigate and describe applications of geometric properties (e.g., properties of triangles, quadrilaterals, and circles) in the real world.

Geometric Relationships

By the end of Grade 8, students will:

– determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials, geoboard), relationships among area, perimeter, corresponding side lengths, and corresponding angles of similar shapes (*Sample problem:* Construct three similar rectangles, using grid paper or a geoboard, and compare the perimeters and areas of the rectangles.);

– determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials, protractor) and strategies (e.g., paper folding), the angle relationships for intersecting lines and for parallel lines and transversals, and the sum of the angles of a triangle;

– solve angle-relationship problems involving triangles (e.g., finding interior angles or complementary angles), intersecting lines (e.g., finding supplementary angles or opposite angles), and parallel lines and transversals (e.g., finding alternate angles or corresponding angles);

– determine the Pythagorean relationship, through investigation using a variety of tools (e.g., dynamic geometry software; paper and scissors; geoboard) and strategies;

– solve problems involving right triangles geometrically, using the Pythagorean relationship;

– determine, through investigation using concrete materials, the relationship between the numbers of faces, edges, and vertices of a polyhedron (i.e., *number of faces* + *number of vertices* = *number of edges* + 2) (*Sample problem:* Use Polydrons and/or paper nets to construct the five Platonic solids [i.e., tetrahedron, cube, octahedron, dodecahedron, icosahedron], and compare the sum of the numbers of faces and vertices to the number of edges for each solid.).

Location and Movement

By the end of Grade 8, students will:

– graph the image of a point, or set of points, on the Cartesian coordinate plane after applying a transformation to the original point(s) (i.e., translation; reflection in the *x*-axis, the *y*-axis, or the angle bisector of the axes that passes through the first and third quadrants; rotation of 90°, 180°, or 270° about the origin);

– identify, through investigation, real-world movements that are translations, reflections, and rotations.

Grade 8: Patterning and Algebra

Overall Expectations

By the end of Grade 8, students will:

- represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations;
- model linear relationships graphically and algebraically, and solve and verify algebraic equations, using a variety of strategies, including inspection, guess and check, and using a "balance" model.

Specific Expectations

Patterns and Relationships

By the end of Grade 8, students will:

– represent, through investigation with concrete materials, the general term of a linear pattern, using one or more algebraic expressions (e.g., "Using toothpicks, I noticed that 1 square needs 4 toothpicks, 2 connected squares need 7 toothpicks, and 3 connected squares need 10 toothpicks. I think that for n connected squares I will need $4 + 3(n – 1)$ toothpicks, because the number of toothpicks keeps going up by 3 and I started with 4 toothpicks. Or, if I think of starting with 1 toothpick and adding 3 toothpicks at a time, the pattern can be represented as $1 + 3n$.");

– represent linear patterns graphically (i.e., make a table of values that shows the term number and the term, and plot the coordinates on a graph), using a variety of tools (e.g., graph paper, calculators, dynamic statistical software);

– determine a term, given its term number, in a linear pattern that is represented by a graph or an algebraic equation (*Sample problem:* Given the graph that represents the pattern 1, 3, 5, 7, …, find the 10th term. Given the algebraic equation that represents the pattern, $t = 2n – 1$, find the 100th term.).

Variables, Expressions, and Equations

By the end of Grade 8, students will:

– describe different ways in which algebra can be used in real-life situations (e.g., the value of $5 bills and toonies placed in a envelope for fund raising can be represented by the equation $v = 5f + 2t$);

– model linear relationships using tables of values, graphs, and equations (e.g., the sequence 2, 3, 4, 5, 6, … can be represented by the equation $t = n + 1$, where n represents the term number and t represents the term), through investigation using a variety of tools (e.g., algebra tiles, pattern blocks, connecting cubes, base ten materials) (*Sample problem:* Leah put $350 in a bank certificate that pays 4% simple interest each year. Make a table of values to show how much the bank certificate is worth after five years, using base ten materials to help you. Represent the relationship using an equation.);

– translate statements describing mathematical relationships into algebraic expressions and equations (e.g., for a collection of triangles, the total number of sides is equal to three times the number of triangles or $s = 3n$);

– evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables (e.g., evaluate $3x + 4y = 2z$, where $x = \frac{1}{2}$, $y = 0.6$, and $z = –1$);

– make connections between solving equations and determining the term number in a pattern, using the general term (e.g., for the pattern with the general term $2n + 1$, solving the equation $2n + 1 = 17$ tells you the term number when the term is 17);

– solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a “balance” model (*Sample problem:* What is the value of the variable in the equation $30x – 5 = 10$?).

Grade 8: Data Management and Probability

Overall Expectations

By the end of Grade 8, students will:

- collect and organize categorical, discrete, or continuous primary data and secondary data and display the data using charts and graphs, including frequency tables with intervals, histograms, and scatter plots;
- apply a variety of data management tools and strategies to make convincing arguments about data;
- use probability models to make predictions about real-life events.

Specific Expectations

Collection and Organization of Data

By the end of Grade 8, students will:

– collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;

– organize into intervals a set of data that is spread over a broad range (e.g., the age of respondents to a survey may range over 80 years and may be organized into ten-year intervals);

– collect and organize categorical, discrete, or continuous primary data and secondary data (e.g., electronic data from websites such as E-Stat or Census At Schools), and display the data in charts, tables, and graphs (including histograms and scatter plots) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, spreadsheets, dynamic statistical software);

– select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied, including histograms and scatter plots);

– explain the relationship between a census, a representative sample, sample size, and a population (e.g., "I think that in most cases a larger sample size will be more representative of the entire population.").

Data Relationships

By the end of Grade 8, students will:

– read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., election data or temperature data from the newspaper, data from the Internet about lifestyles), presented in charts, tables, and graphs (including frequency tables with intervals, histograms, and scatter plots);

– determine, through investigation, the appropriate measure of central tendency (i.e., mean, median, or mode) needed to compare sets of data (e.g., in hockey, compare heights or masses of players on defence with that of forwards);

– demonstrate an understanding of the appropriate uses of bar graphs and histograms by comparing their characteristics (*Sample problem:* How is a histogram similar to and different from a bar graph? Use examples to support your answer.);

– compare two attributes or characteristics (e.g., height versus arm span), using a scatter plot, and determine whether or not the scatter plot suggests a relationship (*Sample problem:* Create a scatter plot to compare the lengths of the bases of several similar triangles with their areas.);

– identify and describe trends, based on the rate of change of data from tables and graphs, using informal language (e.g., "The steep line going upward on this graph represents rapid growth. The steep line going downward on this other graph represents rapid decline.");

– make inferences and convincing arguments that are based on the analysis of charts, tables, and graphs (*Sample problem:* Use data to make a convincing argument that the environment is becoming increasingly polluted.);

– compare two attributes or characteristics, using a variety of data management tools and strategies (i.e., pose a relevant question, then design an experiment or survey, collect and analyse the data, and draw conclusions) (*Sample problem:* Compare the length and width of different-sized leaves from a maple tree to determine if maple leaves grow proportionally. What generalizations can you make?).

Probability

By the end of Grade 8, students will:

– compare, through investigation, the theoretical probability of an event (i.e., the ratio of the number of ways a favourable outcome can occur compared to the total number of possible outcomes) with experimental probability, and explain why they might differ (*Sample problem:* Toss a fair coin 10 times, record the results, and explain why you might not get the predicted result of 5 heads and 5 tails.);

– determine, through investigation, the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases, using class-generated data and technology-based simulation models (*Sample problem:* Compare the theoretical probability of getting a 6 when tossing a number cube with the experimental probabilities obtained after tossing a number cube once, 10 times, 100 times, and 1000 times.);

– identify the complementary event for a given event, and calculate the theoretical probability that a given event will *not* occur (*Sample problem:* Bingo uses the numbers from 1 to 75. If the numbers are pulled at random, what is the probability that the first number is a multiple of 5? is not a multiple of 5?).

Glossary

The following definitions of terms are intended to help teachers and parents use this document.

acute angle. An angle whose measure is between 0º and 90º.

addition. The operation that represents the sum of two or more numbers. The inverse operation of addition is subtraction.

algebra tiles. Learning tools that can be used to model operations involving integers, expressions, and equations. Each tile represents a particular term, such as 1, x, or x^2.

algorithm. A systematic procedure for carrying out a computation. For example, the addition algorithm is a set of rules for finding the sum of two or more numbers.

alternate angles. Two angles on opposite sides of a transversal when it crosses two lines. The angles are equal when the lines are parallel. The angles form one of these patterns: Z, Σ.

analogue clock. A timepiece that measures the time through the position of its hands.

angle. A shape formed by two rays or two line segments with a common endpoint. *See also* **vertex**.

area model. A diagrammatic representation that uses area to demonstrate other mathematical concepts. In an area model for multiplication, for example, the length and width of a rectangle represent the factors, and the area of the rectangle represents the product. The diagram shows the use of an area model to represent 26 x 14.

	20	6
10	200	60
4	80	24

26 x 14 = 200 + 60 + 80 + 24
= 364

array. A rectangular arrangement of objects into rows and columns, used to represent multiplication (e.g., 5 x 3 can be represented by 15 objects arranged into 5 columns and 3 rows).

associative property. A property of addition and multiplication that allows the numbers being added or multiplied to be regrouped without changing the outcome of the operations. For example, (7 + 9) + 1 = 7 + (9 + 1) and (7 x 4) x 5 = 7 x (4 x 5) In general, $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$. Using the associative property can simplify computation. This property does not generally hold for subtraction or division.

attribute. A quantitative or qualitative characteristic of a shape, an object, or an occurrence; for example, colour, size, thickness, or number of sides. An attribute may or may not be a property. *See also* **property (geometric)**.

attribute blocks. Learning tools that help students learn about shapes, sorting, patterning, congruence, similarity, geometric properties, and so on. The standard set of attribute blocks (60 blocks) includes five shapes (rectangle, square, circle, triangle, hexagon); each shape comes in three colours (red, yellow, blue), two sizes (large, small), and two thicknesses (thick, thin).

bar graph. *See under* **graph**.

base. *See* **exponential form**.

base ten materials. Learning tools that help students learn a wide variety of concepts in number sense, including place value; the operations (addition, subtraction, multiplication, and division); and fractions and decimals. Sets of base ten materials typically include ones (small cubes called “units”), tens (“rods” or “longs”), hundreds (“flats”), and thousands (large cubes).

benchmark. A number or measurement that is internalized and used as a reference to help judge other numbers or measurements. For example, the width of the tip of the little finger is a common benchmark for one centimetre. Also called *referent*.

bias. An emphasis on characteristics that are not typical of an entire population and that may result in misleading conclusions.

bisector. A line that divides a segment or an angle into two equal parts. A line that divides another line in half and intersects that line at a 90º angle is called a *perpendicular bisector*.

broken-line graph. *See under* **graph**.

capacity. The greatest amount that a container can hold; usually measured in litres or millilitres.

cardinal directions. The four main points of the compass: north, east, south, and west.

Cartesian coordinate grid. *See* **coordinate plane**.

Cartesian plane. *See* **coordinate plane**.

categorical data. Data that can be sorted by type or quality, rather than by measured or counted values. Eye colour and favourite food are examples of categorical data.

census. The collection of data from an entire population.

circle. The points on a plane that are all the same distance from a centre.

circle graph. *See under* **graph**.

clustering. *See under* **estimation strategies**.

coefficient. A factor of a term. In a term that contains a number and a variable (or variables), connected by multiplication, the numerical factor is the numerical coefficient, and the variable factor is the variable coefficient. For example, in $5y$, 5 is the numerical coefficient and y is the variable coefficient.

commutative property. A property of addition and multiplication that allows the numbers to be added or multiplied in any order, without affecting the sum or product of the operation. For example, $2 + 3 = 3 + 2$ and $2 \times 3 = 3 \times 2$. In general, $a + b = b + a$ and $a \times b = b \times a$. Using the commutative property can simplify computation. This property does not generally hold for subtraction or division.

comparative bar graph. *See* **double bar graph** *under* **graph**.

compass. A tool used for drawing arcs and circles.

compatible numbers. Numbers that are easy to compute mentally and can be used to estimate or calculate an answer. Also called *friendly numbers*. *See* **using compatible numbers** *under* **estimation strategies**.

complementary angles. Two angles whose sum is 90º.

complementary events. Two events that have no outcome(s) in common but that account for all possible outcomes of an experiment. For example, rolling an even number and rolling an odd number using a number cube are complementary events. The sum of the probabilities of complementary events is 1. *See also* **event**.

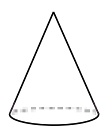
composite number. A number that has factors in addition to itself and 1. For example, the number 8 has four factors: 1, 2, 4, and 8. *See also* **prime number**.

computational strategies. Any of a variety of methods used for performing computations; for example, estimation, mental calculation, student-generated and standard algorithms, and the use of technology (including calculators and computer spreadsheets).

concrete graph. *See under* **graph**.

concrete materials. Objects that students handle and use in constructing or demonstrating their understanding of mathematical concepts and skills. Some examples of concrete materials are base ten blocks, connecting cubes, construction kits, number cubes, games, geoboards, geometric solids, measuring tapes, Miras, pattern blocks, spinners, and tiles. Also called *manipulatives*.

cone. A three-dimensional figure with a circular base and a curved surface that tapers proportionally to an apex.

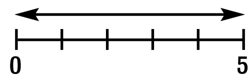


congruent. Having the same size and shape. For example, in two congruent triangles, the three corresponding pairs of sides and the three corresponding pairs of angles are equal.

connecting cubes. Commercially produced learning tools that help students learn about spatial sense, volume, surface area, patterning, and so on. Some connecting cubes attach on only one face, while others attach on any face.

conservation. The property by which something remains the same, despite changes such as physical arrangement. For example, with conservation of number, whether three objects are close together or far apart, the quantity remains the same.

continuous data. Data that can include any numerical value that is represented on a number line and that falls within the range of the data, including decimals and fractions. Continuous data usually represent measurements, such as time, height, and mass. *See also* **discrete data**.



continuous line graph. *See under* **graph.**

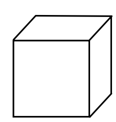
coordinate graph. *See under* **graph**.

coordinate plane. A plane that contains an *x*-axis (horizontal) and a *y*-axis (vertical), which are used to describe the location of a point. Also called *Cartesian coordinate grid* or *Cartesian plane*.

coordinates. An ordered pair used to describe location on a grid or plane. For example, the coordinates (3, 5) describe a location found by moving 3 units to the right and 5 units up from the *origin* (0, 0). *See also* **ordered pair**.

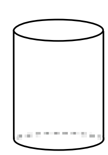
counter-example. An example that shows a general statement to be false.

cube. A right rectangular prism with six congruent square faces. A cube is one of the Platonic solids. Also called a *hexahedron*.



Cuisenaire rods. Commercially produced learning tools that help students learn about fractions, patterning, and so on. This set of rectangular rods of different lengths, in which each length is a different colour, was invented by Georges Cuisenaire (1891-1976), a Belgian schoolteacher.

cylinder. A three-dimensional figure with two congruent, parallel, circular faces and one curved surface.



data. Facts or information. *See also* **categorical data**, **continuous data**, *and* **discrete data**.

database. An organized and sorted list of facts or information; usually generated by a computer.

deductive reasoning. The process of reaching a conclusion by applying arguments that have already been proved and using evidence that is known to be true. Generalized statements are used to prove whether or not specific statements are true.

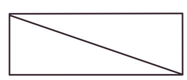
degree. A unit for measuring angles. For example, one full revolution measures 360º.

denominator. The number below the line in a fraction. For example, in $\frac{3}{4}$, the denominator is 4. *See also* **numerator**.

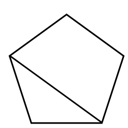
dependent variable. A variable whose value depends on the value of another variable. In graphing, the dependent variable is represented on the vertical axis. *See also* **independent variable**.

diagonal. A line segment joining two vertices of a polygon that are not next to each other (i.e., that are not joined by one side).

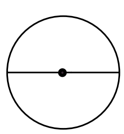
diagonal in a rectangle



diagonal in a pentagon

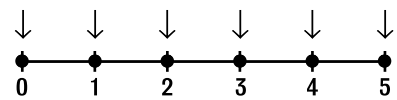


diameter. A line segment that joins two points on a circle and passes through the centre.



dilatation. A transformation that enlarges or reduces a shape by a scale factor to form a similar shape.

discrete data. Data that can include only certain numerical values (often whole numbers) within the range of the data. Discrete data usually represent things that can be counted; for example, the number of times a word is used or the number of students absent. There are gaps between the values. For example, if whole numbers represent the data, as shown in the following diagram, fractional values such as $3\frac{1}{2}$ are not part of the data. *See also* **continuous data**.



displacement. The amount of water displaced by an object placed in it. Measuring the amount of water displaced when an object is completely immersed is a way to find the volume of the object.

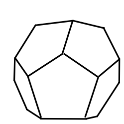
distribution. An arrangement of measurements and related frequencies; for example, a table or graph that shows how many times each score, event, or measurement occurred.

distributive property. The property that allows a number in a multiplication expression to be decomposed into two or more numbers; for example, 51 x 12 = 51 x 10 + 51 x 2. More formally, the distributive property holds that, for three numbers, *a*, *b*, and *c*, *a* x (*b* + *c*) = (*a* x *b*) + (*a* x *c*) and *a* x (*b* – *c*) = (*a* x *b*) – (*a* x *c*); for example, 2 x (4 + 1) = 2 x 4 + 2 x 1 and 2 x (4 – 1) = 2 x 4 – 2 x 1. Multiplication is said to be distributed over addition and subtraction.

division. The operation that represents repeated subtraction or the equal sharing of a quantity. The inverse operation of division is multiplication.

dodecahedron. A polyhedron with 12 faces. The regular dodecahedron is one of the Platonic solids and has faces that are regular pentagons.

regular dodecahedron



double bar graph. *See under* **graph**.

dynamic geometry software. Computer software that allows the user to explore and analyse geometric properties and relationships through dynamic dragging and animations. Uses of the software include plotting points and making graphs on a coordinate system; measuring line segments and angles; constructing and transforming two-dimensional shapes; and creating two-dimensional representations of three-dimensional objects. An example of the software is *The Geometer's Sketchpad*.

dynamic statistical software. Computer software that allows the user to gather, explore, and analyse data through dynamic dragging and animations. Uses of the software include organizing data from existing tables or the Internet, making different types of graphs, and determining measures of central tendency. Examples of the software include *TinkerPlots* and *Fathom*.

equation. A mathematical statement that has equivalent expressions on either side of an equal sign.

equilateral triangle. A triangle with three equal sides.

equivalent fractions. Different representations in fractional notation of the same part of a whole or group; for example, $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$.

equivalent ratios. Ratios that represent the same comparison, and whose fractional forms reduce to the same value; for example, 1:3, 2:6, 3:9, 4:12.

estimation strategies. Mental mathematics strategies used to obtain an approximate answer. Students estimate when an exact answer is not required, and to check the reasonableness of their mathematics work. Some estimation strategies are:

– **clustering.** A strategy used for estimating the sum of numbers that cluster around one particular value. For example, the numbers 42, 47, 56, 55 cluster around 50. So estimate 50 + 50 + 50 + 50 = 200.

– **rounding.** A process of replacing a number by an approximate value of that number. For example, 193 + 428 + 253 can be estimated by rounding to the nearest 100. So estimate 200 + 400 + 300 = 900.

– **using compatible numbers.** A process of identifying and using numbers that can be computed mentally. For example, to estimate 28 ÷ 15, dividing the compatible numbers 30 and 15, or the compatible numbers 28 and 14, results in an estimate of about 2. *See also* **compatible numbers**.

event. A possible outcome, or group of outcomes, of an experiment. For example, rolling an even number on a number cube is an event with three possible outcomes: 2, 4, and 6.

expanded form. A way of writing numbers that shows the value of each digit; for example, 432 = 4 x 100 + 3 x 10 + 2. *See also* **place value**, **standard form**.

experimental probability. The likelihood of an event occurring, determined from experimental results rather than from theoretical reasoning.

exponent. *See* **exponential form**.

exponential form. A representation of a product in which a number called the *base* is multiplied by itself. The *exponent* is the number of times the base appears in the product. For example, 5^4 is in exponential form, where 5 is the base and 4 is the exponent; 5^4 means 5 x 5 x 5 x 5.

expression. A numeric or algebraic representation of a quantity. An expression may include numbers, variables, and operations; for example, 3 + 7, 2*x* – 1.

factors. Natural numbers that divide evenly into a given natural number. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12, because all of these numbers divide evenly into 12. *See also* **multiplication**.

flip. *See* **reflection**.

formula. An equation summarizing a relationship between measureable attributes; for example, for a right prism, *Volume* = *area of base* x *height*.

fraction circles. Learning tools that help students learn about fractions. Common commercially produced fraction circle sets, made of foam or plastic, have circles cut into halves, thirds, fourths, and so on, in different colours.

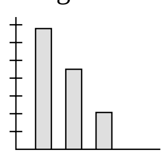
frequency. The number of times an event or outcome occurs.

general term. An algebraic expression that represents any term in a pattern or sequence, based on the term number. For example, in the sequence 2, 4, 6, 8, 10, …, the general term is $2n$. Also called *nth term*.

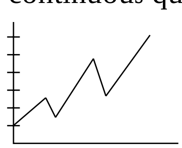
geoboard. A commercially produced learning tool that helps students learn about perimeter, area, fractions, transformations, and so on. A geoboard is a square piece of plastic or wood with pins arranged in a grid or in a circle. Elastics are used to connect the pins to make different shapes.

graph. A visual representation of data. Some types of graphs are:

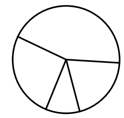
– **bar graph.** A graph consisting of horizontal or vertical bars that represent the frequency of an event or outcome. There are gaps between the bars to reflect the categorical or discrete nature of the data.



– **broken-line graph.** A graph formed by line segments that join points representing the data. The horizontal axis represents discrete quantities such as months or years, whereas the vertical axis can represent continuous quantities.



– **circle graph.** A graph in which a circle is used to display categorical data, through the division of the circle proportionally to represent each category.

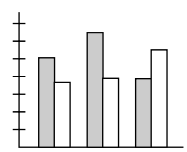


– **concrete graph.** A graph on which real objects are used to represent pieces of information; for example, coloured candy directly placed on a template of a bar graph.

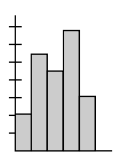
– **continuous line graph.** A graph that consists of an unbroken line and in which both axes represent continuous quantities, such as distance and time.

– **coordinate graph.** A graph that has data points represented as ordered pairs on a grid; for example, (4, 3). *See also* **ordered pair**.

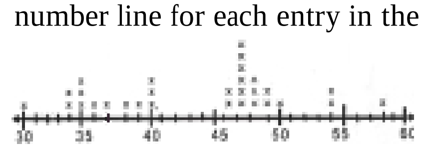
– **double bar graph.** A graph that combines two bar graphs to compare two aspects of the data in related contexts; for example, comparing the populations of males and females in a school in different years. Also called *comparative bar graph*.



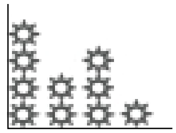
– **histogram.** A type of bar graph in which each bar represents a range of values, and the data are continuous. No spaces are left between the bars, to reflect the continuous nature of the data.



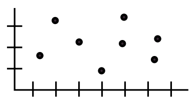
– **line plot.** A graph that shows a mark (usually an "X") above a value on the number line for each entry in the data set.



– **pictograph.** A graph that uses pictures or symbols to compare frequencies.



– **scatter plot.** A graph designed to show a relationship between corresponding numbers from two sets of data measurements associated with a single object or event; for example, a graph of data about marks and the corresponding amount of study time. Drawing a scatter plot involves plotting ordered pairs on a coordinate grid. Also called *scatter diagram*.



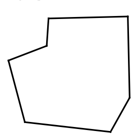
– **stem-and-leaf plot.** An organization of data into categories based on place values. The plot allows easy identification of the greatest, least, and median values in a set of data. The following stem-and-leaf plot represents these test results: 72, 64, 68, 82, 75, 74, 68, 70, 92, 84, 77, 59, 77, 70, 85.

5	9
6	4, 8, 8
7	0, 0, 2, 4, 5, 7, 7
8	2, 4, 5
9	2

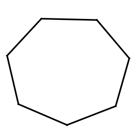
greatest common factor. The largest factor that two or more numbers have in common. For example, the greatest common factor of 16 and 24 is 8.

heptagon. A polygon with seven sides.

heptagon

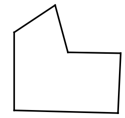


regular heptagon

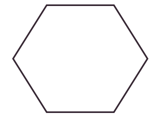


hexagon. A polygon with six sides.

hexagon



regular hexagon

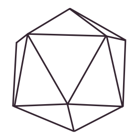


histogram. *See under* **graph**.

hundreds chart. A 10 x 10 table or chart with each cell containing a natural number from 1 to 100 arranged in order.

icosahedron. A polyhedron with 20 faces. The regular icosahedron is one of the Platonic solids and has faces that are equilateral triangles.

regular icosahedron



improper fraction. A fraction whose numerator is greater than its denominator; for example, $\frac{12}{5}$.

independent events. Two or more events where one does not affect the probability of the other(s); for example, rolling a 6 on a number cube and drawing a red card from a deck.

independent variable. A variable for which values are freely chosen and do not depend on the values of other variables. In graphing, the independent variable is represented on the horizontal axis. *See also* **dependent variable**.

inductive reasoning. The process of reaching conclusions based on observations of patterns. Specific statements and observations are used to make generalizations.

inference. A conclusion drawn from any method of reasoning. *See also* **deductive reasoning**, **inductive reasoning**.

integer. Any one of the numbers … , –4, –3, –2, –1, 0, 1, 2, 3, 4, ….

intersecting lines. Lines that cross each other and that have exactly one point in common, the point of intersection.

interval. The set of points or the set of numbers that exist between two given endpoints. The endpoints may or may not be included in the interval. For example, test score data can be organized into intervals such as 65 – 69, 70 – 74, 75 – 79, and so on.

inverse operations. Two operations that "undo" or "reverse" each other. For example, for any number, adding 7 and then subtracting 7 gives the original number. The subtraction undoes or reverses the addition.

irrational number. A number that cannot be represented as a terminating or repeating decimal; for example, $\sqrt{5}$, π.

irregular polygon. A polygon that does not have all sides and all angles equal. *See also* **regular polygon**.

isometric dot paper. Dot paper used for creating perspective drawings of three-dimensional figures. The dots are formed by the vertices of equilateral triangles. Also called *triangular dot paper* or *triangle dot paper*.

isosceles triangle. A triangle that has two sides of equal length.

least common multiple. The smallest number that two numbers can divide into evenly. For example, 30 is the least common multiple of 10 and 15.

line of symmetry. A line that divides a shape into two congruent parts that can be matched by folding the shape in half.

linear dimension. A measurement of one linear attribute; that is, distance, length, width, height, or depth.

linear equation. An algebraic representation of a linear relationship. The relationship involves one or more first-degree variable terms; for example, $y = 2x - 1$; $2x + 3y = 5$; $y = 3$. The graph of a linear equation is a straight line.

linear relationship. A relationship between two measurable quantities that appears as a straight line when graphed on a coordinate system.

line plot. *See under* **graph**.

magnitude. An attribute relating to size or quantity.

manipulatives. *See* **concrete materials**.

many-to-one correspondence. The correspondence of more than one object to a single symbol or picture. For example, on a pictograph, five cars can be represented by one sticker. *See also* **one-to-one correspondence**.

mass. The amount of matter in an object; usually measured in grams or kilograms.

mathematical communication. The process through which mathematical thinking is shared. Students communicate by talking, drawing pictures, drawing diagrams, writing journals, charting, dramatizing, building with concrete materials, and using symbolic language (e.g., 2, =).

mathematical concept. A connection of mathematical ideas that provides a deep understanding of mathematics. Students develop their understanding of mathematical concepts through rich problem-solving experiences.

mathematical conventions. Agreed-upon rules or symbols that make the communication of mathematical ideas easier.

mathematical language. The conventions, vocabulary, and terminology of mathematics. Mathematical language may be used in oral, visual, or written forms. Some types of mathematical language are:

- terminology (e.g., factor, pictograph, tetrahedron);
- visual representations (e.g., 2 x 3 array, parallelogram, tree diagram);
- symbols, including numbers (e.g., 2, $\frac{1}{4}$), operations [e.g., 3 x 8 = (3 x 4) + (3 x 4)], and signs (e.g., =).

mathematical procedures. The operations, mechanics, algorithms, and calculations used to solve problems.

mean. One measure of central tendency. The mean of a set of numbers is found by dividing the sum of the numbers by the number of numbers in the set. For example, the mean of 10, 20, and 60 is (10 + 20 + 60) ÷ 3 = 30. A change in the data produces a change in the mean, similar to the way in which changing the load on a lever affects the position of the fulcrum if balance is maintained. *See also* **measure of central tendency**.

measure of central tendency. A value that summarizes a whole set of data; for example, the mean, the median, or the mode. A measure of central tendency represents the approximate centre of a set of data. Also called *central measure*. *See also* **mean**, **median**, **mode**.

median. The middle value in a set of values arranged in order. For example, 14 is the median for the set of numbers 7, 9, 14, 21, 39. If there is an even number of numbers, the median is the average of the two middle numbers. For example, 11 is the median of 5, 10, 12, and 28. *See also* **measure of central tendency**.

mental strategies. Ways of computing mentally, with or without the support of paper and pencil. *See also* **estimation strategies**.

Mira. A commercially produced transparent mirror. This learning tool is used in geometry to locate reflection lines, reflection images, and lines of symmetry, and to determine congruency and line symmetry.

mixed number. A number that is composed of a whole number and a fraction; for example, $8\frac{1}{4}$.

mode. The value that occurs most often in a set of data. For example, in a set of data with the values 3, 5, 6, 5, 6, 5, 4, 5, the mode is 5. *See also* **measure of central tendency**.

modelling. The process of describing a relationship using mathematical or physical representations.

monomial. An algebraic expression with one term; for example, $2x$ or $5xy$.

multiple. The product of a given whole number multiplied by any other whole number except 1. For example, 4, 8, 12, … are multiples of 4.

multiplication. An operation that represents repeated addition, the combining of equal groups, or an array. The multiplication of *factors* gives a product. For example, 4 and 5 are factors of 20 because 4 x 5 = 20. The inverse operation of multiplication is division. *See also* **factors**.

multi-step problem. A problem that is solved by making at least two calculations. For example, shoppers who want to find out how much money they will have left after some purchases can follow these steps:

Step 1 Add the costs of all items to be purchased (subtotal).

Step 2 Multiply the sum of the purchases by the percent of tax.

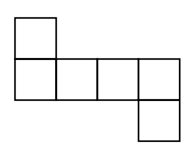
Step 3 Add the tax to the sum of the purchases (grand total).

Step 4 Subtract the grand total from the shopper's original amount of money.

natural numbers. The counting numbers 1, 2, 3, 4, ….

net. A pattern that can be folded to make a three-dimensional figure.

a net of a cube

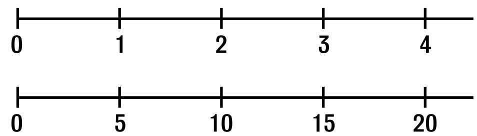


non-standard units. Common objects used as measurement units; for example, paper clips, cubes, and hand spans. Non-standard units are used in the early development of measurement concepts.

***n*th term.** *See* **general term**.

number cube. A learning tool that can help students learn a variety of concepts, including counting, operations, and probability. A number cube is a small cube that is typically made of plastic or wood. The faces are marked with different numerals or, in the case of dice, with different numbers of dots, usually representing the whole numbers from 1 to 6.

number line. A line that represents a set of numbers using a set of points. The increments on the number line reflect the scale.

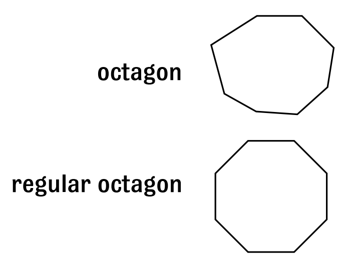


number operations. Procedures for combining numbers. The procedures include addition, subtraction, multiplication, and division.

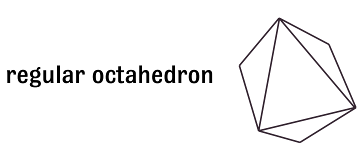
numerator. The number above the line in a fraction. For example, in $\frac{3}{4}$, the numerator is 3. *See also* **denominator**.

obtuse angle. An angle that measures more than 90º and less than 180º.

octagon. A polygon with eight sides.



octahedron. A polyhedron with eight faces. The regular octahedron is one of the Platonic solids and has faces that are equilateral triangles.



one-to-one correspondence. The correspondence of one object to one symbol or picture. In counting, one-to-one correspondence is the idea that each object being counted must be given one count and only one count. *See also* **many-to-one correspondence**.

order of operations. A convention or rule used to simplify expressions. The acronym BEDMAS is often used to describe the order:

- brackets
- exponents
- division or
- multiplication, whichever comes first
- addition or
- subtraction, whichever comes first

order of rotational symmetry. The number of times the position of a shape coincides with its original position during one complete rotation about its centre. For example, a square has rotational symmetry of order 4. *See also* **rotational symmetry**.

ordered pair. Two numbers, in order, that are used to describe the location of a point on a plane, relative to a point of origin (0,0); for example, (2, 6). On a coordinate plane, the first number is the horizontal coordinate of a point, and the second is the vertical coordinate of the point. *See also* **coordinates**.

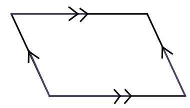
ordinal number. A number that shows relative position or place; for example, first, second, third, fourth.

outlier. A data point that is separated from the rest of the points on a graph. An outlier *may* indicate something unusual in the situation being studied, or an error in the data collection process.

pan balance. A device consisting of two pans supported at opposite ends of a balance beam. A pan balance is used to compare and measure masses of objects. Also called *double-pan balance*.

parallel lines. Lines in the same plane that do not intersect.

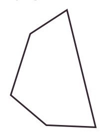
parallelogram. A quadrilateral whose opposite sides are parallel.



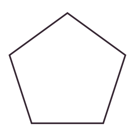
pattern blocks. Commercially produced learning tools that help students learn about shapes, patterning, fractions, angles, and so on. Standard sets include: green triangles; orange squares; tan rhombuses and larger blue rhombuses; red trapezoids; yellow hexagons.

pentagon. A polygon with five sides.

pentagon



regular pentagon



percent. A ratio expressed using the percent symbol, %. Percent means "out of a hundred". For example, 30% means 30 out of 100. A percent can be represented by a fraction with a denominator of 100; for example, $30\% = \frac{30}{100}$.

perfect square. A number that can be expressed as the product of two identical natural numbers. For example, $9 = 3 \times 3$; thus 9 is a perfect square.

perimeter. The length of the boundary of a shape, or the distance around a shape. For example, the perimeter of a rectangle is the sum of its side lengths; the perimeter of a circle is its circumference.

perpendicular lines. Two lines in the same plane that intersect at a 90º angle.

pictograph. *See under* **graph**.

place value. The value of a digit that appears in a number. The value depends on the position or place in which the digit appears in the number. For example, in the number 5473, the digit 5 is in the thousands place and represents 5000; the digit 7 is in the tens place and represents 70.

Polydrons. Commercially produced learning tools that help students learn about the geometric properties, surface areas, and volumes of three-dimensional figures. Polydrons are plastic connecting shapes used to construct three-dimensional figures and their nets.

polygon. A closed shape formed by three or more line segments; for example, triangle, quadrilateral, pentagon, octagon.

polyhedron. A three-dimensional figure that has polygons as faces.

population. The total number of individuals or objects that fit a particular description; for example, salmon in Lake Ontario.

power. A number written in exponential form; a shorter way of writing repeated multiplication. For example, 10^2 and 2^6 are powers. *See also* **exponential form**.

Power Polygons. Commercially produced learning tools that help students learn about shapes and the relationships between their areas. Power Polygons are transparent plastic shapes that include triangles, parallelograms, trapezoids, rectangles, and so on.

primary data. Information that is collected directly or first-hand; for example, observations and measurements collected directly by students through surveys and experiments. Also called *first-hand data* or *primary-source data*. *See also* **secondary data**.

prime factorization. An expression showing a composite number as the product of its prime factors. The prime factorization for 24 is $2 \times 2 \times 2 \times 3$.

prime number. A whole number greater than 1 that has only two factors, itself and 1. For example, the only factors of 7 are 7 and 1. *See also* **composite number**.

prism. A three-dimensional figure with two parallel and congruent bases. A prism is named by the shape of its bases; for example, rectangular prism, triangular prism.

probability. A number from 0 to 1 that shows how likely it is that an event will happen.

product. *See* **multiplication**.

proper fraction. A fraction whose numerator is smaller than its denominator; for example, $\frac{2}{3}$.

property (geometric). An attribute that remains the same for a class of objects or shapes. A property of any parallelogram, for example, is that its opposite sides are congruent. *See also* **attribute**.

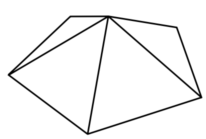
proportion. An equation showing equivalent ratios in fraction form; for example, $\frac{2}{3} = \frac{6}{9}$.

proportional reasoning. Reasoning based on the use of equal ratios.

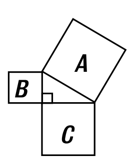
protractor. A tool for measuring angles.

pyramid. A polyhedron whose base is a polygon and whose other faces are triangles that meet at a common vertex.

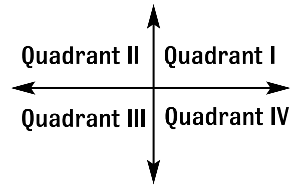
pentagonal pyramid



Pythagorean relationship. The relationship that, for a right triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides. In the diagram, $A = B + C$.



quadrant. One quarter of the Cartesian plane, bounded by the coordinate axes.



quadrilateral. A polygon with four sides.

radius. A line segment whose endpoints are the centre of a circle and a point on the circle. The radius is half the diameter.

range. The difference between the highest and lowest numbers in a group of numbers or set of data. For example, in the data set 8, 32, 15, 10, the range is 24, that is, 32 – 8.

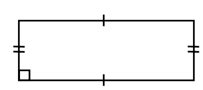
rate. A comparison, or a type of ratio, of two measurements with different units, such as distance and time; for example, 100 km/h, 10 kg/m^3, 20 L/100 km.

rate of change. A change in one quantity relative to the change in another quantity. For example, for a 10 km walk completed in 2 h at a steady pace, the rate of change is $\frac{10\text{ km}}{2\text{ h}}$ or 5 km/h.

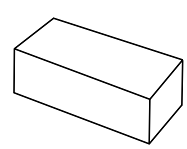
ratio. A comparison of quantities with the same units. A ratio can be expressed in ratio form or in fraction form; for example, 3:4 or $\frac{3}{4}$.

rational number. A number that can be expressed as a fraction in which the denominator is not 0. *See also* **irrational number**.

rectangle. A quadrilateral in which opposite sides are equal, and all interior angles are right angles.



rectangular prism. A prism with opposite congruent rectangular faces.

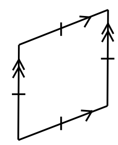


reflection. A transformation that flips a shape over an axis to form a congruent shape. A *reflection image* is the mirror image that results from a reflection. Also called *flip*.

regular polygon. A closed figure in which all sides are equal and all angles are equal. *See also* **irregular polygon**.

relative frequency. The frequency of a particular outcome or event expressed as a percent of the total number of outcomes. *See also* **frequency**.

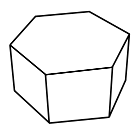
rhombus. A parallelogram with equal sides. Sometimes called a *diamond*.



right angle. An angle that measures 90º.

right prism. A prism whose rectangular faces are perpendicular to its congruent bases.

hexagonal right prism



rotation. A transformation that turns a shape about a fixed point to form a congruent shape. A *rotation image* is the result of a rotation. Also called *turn*.

rotational symmetry. A geometric property of a shape whose position coincides with its original position after a rotation of less than 360º about its centre. For example, the position of a square coincides with its original position after a $\frac{1}{4}$ turn, a $\frac{1}{2}$ turn, and a $\frac{3}{4}$ turn, as well as after a full turn, so a square has rotational symmetry. *See also* **order of rotational symmetry**.

rounding. *See under* **estimation strategies**.

sample. A representative group chosen from a population and examined in order to make predictions about the population.

scale (on a graph). A sequence of numbers associated with marks that subdivide an axis. An appropriate scale is chosen to ensure that all data are represented on the graph.

scale drawing. A drawing in which the lengths are proportionally reduced or enlarged from actual lengths.

scalene triangle. A triangle with three sides of different lengths.

scatter plot. *See under* **graph**.

secondary data. Information that is not collected first-hand; for example, data from a magazine, a newspaper, a government document, or a database. Also called *second-hand data* or *secondary-source data*. *See also* **primary data**.

sequence. A pattern of numbers that are connected by some rule; for example, 3, 5, 7, 9, ….

shape of data. The shape of a graph that represents the distribution of a set of data. The shape of data may or may not be symmetrical.

SI. The international system of measurement units; for example, centimetre, kilogram. (From the French *Système International d'Unités*.)

similar. Having the same shape but not always the same size. If one shape is similar to another shape, there exists a dilatation that will transform the first shape into the second shape.

simple probability experiment. An experiment with the same possible outcomes each time it is repeated, but for which no single outcome is predictable; for example, tossing a coin, rolling a number cube.

simulation. A probability experiment with the same number of outcomes and corresponding probabilities as the situation it represents. For example, tossing a coin could be a simulation of whether the next person you meet will be a male or a female.

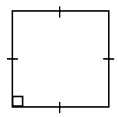
skeleton. A model that shows only the edges and vertices of a three-dimensional figure.

slide. *See* **translation**.

sphere. A perfectly round ball, such that every point on the surface of the sphere is the same distance from the centre of the sphere.

spreadsheet. A tool that helps to organize information using rows and columns.

square. A rectangle with four equal sides and four right angles.



square root of a number. A factor that, when multiplied by itself, equals the number. For example, 3 is a square root of 9, because 3 x 3 = 9.

standard form. A way of writing a number in which each digit has a place value according to its position in relation to the other digits. For example, 7856 is in standard form. *See also* **place value**, **expanded form**.

stem-and-leaf plot. *See under* **graph**.

straight angle. An angle that measures 180°.

subtraction. The operation that represents the difference between two numbers. The inverse operation of subtraction is addition.

supplementary angles. Two angles whose sum is 180°.

surface area. The total area of the surface of a three-dimensional object.

survey. A record of observations gathered from a sample of a population. For example, observations may be gathered and recorded by asking people questions or interviewing them.

symbol. *See under* **mathematical language**.

symmetry. The geometric property of being balanced about a point, a line, or a plane. *See also* **line of symmetry**, **rotational symmetry**.

systematic counting. A process used as a check so that no event or outcome is counted twice.

table. An orderly arrangement of facts set out for easy reference; for example, an arrangement of numerical values in vertical columns and horizontal rows.

tally chart. A chart that uses tally marks to count data and record frequencies.

tangram. A Chinese puzzle made from a square cut into seven pieces: two large triangles, one medium-sized triangle, two small triangles, one square, and one parallelogram.

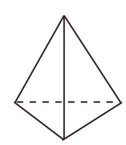
ten frame. A 2 x 5 array in which students place counters or dots to show numbers to 10.

term. Each of the quantities constituting a ratio, a sum or difference, or an algebraic expression. For example, in the ratio 3:5, 3 and 5 are both terms; in the algebraic expression $3x + 2y$, $3x$ and $2y$ are both terms.

tessellation. A tiling pattern in which shapes are fitted together with no gaps or overlaps. A regular tessellation uses congruent shapes. *See also* **tiling**.

tetrahedron. A polyhedron with four faces. A regular tetrahedron is one of the Platonic solids and has faces that are equilateral triangles.

regular tetrahedron



theoretical probability. A mathematical calculation of the chances that an event will happen in theory; if all outcomes are equally likely, it is calculated as the number of favourable outcomes divided by the total number of possible outcomes.

tiling. The process of using repeated shapes, which may or may not be congruent, to cover a region completely. *See also* **tessellation**.

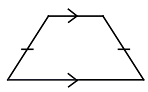
time line. A number line on which the numbers represent time values, such as numbers of days, months, or years.

transformation. A change in a figure that results in a different position, orientation, or size. The transformations include the *translation* (slide), *reflection* (flip), *rotation* (turn), and *dilatation* (reduction or enlargement). *See also* **dilatation**, **reflection**, **rotation**, **translation**.

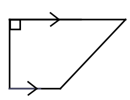
translation. A transformation that moves every point on a shape the same distance, in the same direction, to form a congruent shape. A *translation image* is the result of a translation. Also called *slide*.

trapezoid. A quadrilateral with one pair of parallel sides.

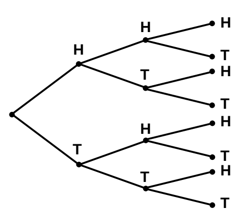
isosceles trapezoid



right trapezoid



tree diagram. A branching diagram that shows all possible combinations or outcomes for two or more independent events. The following tree diagram shows the possible outcomes when three coins are tossed.



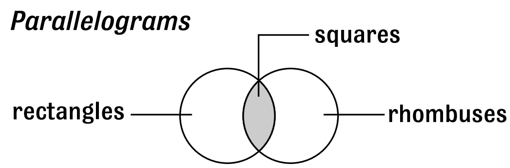
triangle. A polygon with three sides.

turn. *See* **rotation**.

unit rate. A rate that, when expressed as a ratio, has a second term that is one unit. For example, travelling 120 km in 2 h gives a unit rate of 60 km/h or 60 km:1 h.

variable. A letter or symbol used to represent an unknown quantity, a changing value, or an unspecified number (e.g., $a \times b = b \times a$).

Venn diagram. A diagram consisting of overlapping and/or nested shapes used to show what two or more sets have and do not have in common.



vertex. The common endpoint of the two line segments or rays of an angle. *See also* **angle**.

volume. The amount of space occupied by an object; measured in cubic units, such as cubic centimetres.

whole number. Any one of the numbers 0, 1, 2, 3, 4, ….

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